ON THE USE OF INTEGRAL EQUATIONS IN WATER WAVE PROBLEMS.

by

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Consider a long circular cylinder which is immersed in an incompressible inviscid fluid so that its axis lies parallel to the plane of the fluid's free-surface. If this cylinder is forced to undergo a periodic heaving motion of small amplitude and we assume that the fluid motion is irrotational, then the fluid motion may be described in terms of a potential which satisfies Laplace's equation in the fluid, the linearized free-surface condition on the mean level of the fluid and a normal velocity condition on the mean position of the cylinder. When this cylinder is half immersed the resulting problem may be solved using the method of infinite determinants. This problem may also be solved using the theory of integral equations. In this second method the potential is written as a distribution of wave sources of unknown strength. We find that this unknown strength satisfies a Fredholm integral equation of the second kind

$$\mu(s) + \lambda \int_S \mu(t)\, K(s,t)\, dt = f(s) \qquad (1)$$

where $\mu(s)$ is the unknown strength, $K(s,t)$ is the kernel of the equation and $f(s)$ is a known function. By using the method of Fredholm determinants we can show that this integral equation has a solution except for a discrete set of irregular values of λ. (Methods also exist for removing these irregular values). If the cylinder is, say, more than half immersed then neither of these methods apply: the geometry is not suitable for the method of inifinite determinants and representing the potential as a Source distribution leads to an integral equation of the form

$$a(s)\mu_1(s) + \lambda \int_S \mu_1(t)\, K_1(s,t)\, dt = F_1(s) \qquad (II)$$

where $K_1(s,t)$ is singular when s and t simultaneously take values corresponding to the point where the cylinder meets the free-surface. The singular nature of $K_1(s,t)$ and the presence of $a(s)$ means that we can no longer use the theory of Fredholm determinants to ensure the existence of a solution to (II). Similar comments apply to any body which meets the free surface at an angle other than 90°. In this work we shall see how by an appropriate modification of our original source potential we can avoid the difficulties outlined above and derive an integral equation to which the method of Fredholm determinants will apply.

(It should be mentioned here that although the integral equation (II) does not admit to the usual Fredholm theory, it has been shown by Kuznetsov and Maz'ya (1974) that the results of this theory do still in fact hold. The proof of this assertion relies on some very deep results from functional analysis and is very terminal: Kuznetsov and Maz'ya prove the existence of a solution to equation (II) by considering the set of generalized solutions to the potential problem which lie in some appropriate Sobelev space. There is found to be only one generalized solution of the problem which is then shown to be the classical solution we are seeking. The question of how to solve the integral equation (II) is left unresolved).

The idea behind our treatment of the problem lies in choosing a second surface which is inside the cylinder and meets it with the same slope but different curvature at the free-surface. We shall see that by taking a distribution of sources over this second surface in addition to one over the wetted cylinder surface it is possible to derive an integral equation of the form (I) with a continuous kernel. As commented above this is equivalent to modifying the source potential in our original distribution and suggests that the usual form of the source potential

$$G(x,y;\xi,\eta) = \tfrac{1}{2} \log \frac{(x-\xi)^2 + (y-\eta)^2}{(x-\xi)^2+(y+\eta)^2}$$

$$- 2 \oint_0^\infty e^{-u(y+\eta)} \cos u\,(x-\xi)\, \frac{du}{u-K}$$

is not the most appropriate choice for problems involving bodies which meet the free surface at angle other than 90°.

Finally this method will be used to calculate added mass and damping coefficients for the more and less than half immersed circular cylinder undergoing a forced heaving motion.

REFERENCE

N.G. Kuznetsov and V.G. Maz'ya, (1974), Problem concerning steady-state oscillations of a layer of fluid in the presence of an obstacle. Sov. Phys. Dokl., Vol. 19, No. 6.

Discussion

Papanikolaou: It seems to me that your added-mass limit is the case of zero clearance between the top of the cylinder and the free surface in the case of small frequencies tending to zero, becomes one, that is it tends to the solution of the infinite fluid problem. How can you explain the null effect of the condition $\phi_y = 0$ for $\theta = 0$? One might calculate the added-mass of surface piercing cylinders by a mixed source-dipole distribution method in a satisfactory manner despite certain difficulties (see for example, Takagi, Berkely Workshop, 1983)

Walton: The long-wave asymptotics of the added-mass coefficient in this problem will take the form

$$A \ell n Ka + O(1)$$

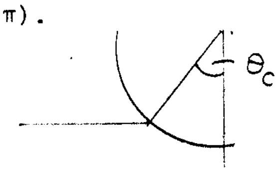
where A is proportional to (water-plane area)^2/(volume) for the body. Since the water-plane area for the nearly submerged body is so small, it is possible that the order one term will dominate until Ka becomes very small, that is smaller than 0.01.

Simon: It is easy to show from the work of Simon & Hulme (1985), extended to the 2-D case, that, for the partially submerged cylinder,

$$A \sim -f(\theta_c)\{\ell n(Ka) + \gamma\} + O(1) \quad \text{and} \quad B \to \pi f(\theta_c)$$

as $Ka \to 0$, where

$$f(\theta_c) = \frac{8}{\pi} \frac{\sin^2\theta_c}{2\theta_c - \sin(2\theta_c)} \quad (0 < \theta_c < \pi).$$



This function and the corresponding values for A & B at Ka = .01 are tabulated below:

θ_c/π	$f(\theta_c)$	A(.01)	B(.01)
.1	6.00	24.17	18.85
.2	2.88	11.60	9.04
.3	1.78	7.19	5.61
.4	1.20	4.82	3.76
(semi-submerged) .5	0.81	3.26	2.55
.6	0.53	2.13	1.66
.7	0.31	1.26	0.98
.8	0.15	0.59	0.46
.9	0.04	0.15	0.12

Disagreement with Walton's results for $\theta_c \to \pi$ may not be significant, as the above tabulated asymptotic result for A(Ka) does not include the O(1) term.

Reference: "The Radiation and Scattering of Long Water Waves" Symposium on Hydrodynamics of Ocean Wave-Energy Utilization IUTAM, Lisbon, Portugal 1985.

X.J.Wu:

1. Generally speaking, numerical computation experiences indicate that the integral equation yields unique solutions except at irregular frequencies and the discretised matrix equation produces convergent solutions even if the intersecting angle of the body wall and the free-surface is not 90°.

2. A problem arises from the intersecting point when a concaved body section intersects the free surface in a small angle since there exists a logarithmic singularity. Such a problem has been studied by Haraguchi and Ohmatsu ("On the improved solution of the oscillation problem on non-wall sided floating bodies...", Trans. West-Japan Soc. Naval Archi., No. 66, Aug. 1983, pp.9-23) and followed by Takagi et al (Workshop Ship and Platform Motions, Berkeley, 1983).

3. This singularity problem may be eliminated by some practical modifications, for example, Haraguchi proposed a method rather similar to that discussed by Walton but adopting additional elements on the free surface connected with the intersecting points. They investigated a circular section in various draft values and found their methods effective.

4. From the physical point of view, it is not reasonable to apply a linear wave theory to a very shallow layer of flow above the body wall slope because the non-linear effect may be dominant.

5. Thus one can only rely on comparisons with experimental data. In comparison with the model testing data Takagi et al found that
(a) Haraguchi's computational results considerably differ from the experimental measurements,
(b) a modified Green function technique incorporating with a slightly changed geometry by substituting the intersecting part with a small arc may give more satisfactory answer.

6. With these points in mind, there may be no numerical problems to apply existing numerical techniques and computational programmes to arbitrary body sectional geometries (X.J. Wu, CADMO 86, Sep. 1986, pp.223-235, Springer-Verlag).

7. In our computational experience additional ill-conditioning frequencies have been found apart from the two identified groups (i.e. the resonant frequencies due to the exterior water waves and the irregular frequencies related to the interior problem). The reasoning is not clear and these seen no effect on the calculated results.