The diffraction of water waves by a circular cylinder in a channel

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Abstract

A uniform vertical circular cylinder is placed at the centre of a wave flume in the presence of a regular incident wavetrain. The depth dependence in the velocity potential can be isolated and as the time dependence is harmonic the problem is essentially reduced to a two-dimensional one involving the determination of a potential function. This potential function satisfies the Helmholtz equation in the fluid domain, together with boundary conditions on the cylinder and channel walls.

Similar diffraction problems are also found in microwave transmission theory. Most of the early work was restricted to electromagnetic wave-guides and the method of solution was generally dependent upon the availability of computational facilities. The first solution was obtained by Schwinger (outlined in Marcuwitz (1951) and given in more detail in Schwinger and Saxon (1968)), using a variational approach to obtain an approximate solution. Further important solutions were obtained by Suzuki (1955) and Nielsen (1969); the first assumes the cylinder radius a to be small relative to the channel half-width b (a/b << 1) and the second employs a subdivision of the domain into three separate regions which are linked by a numerical matching scheme.

The difference between the waveguide and water wave application is essentially one of comparison. In electromagnetic applications, interest is directed to the disruption of the propagating modes caused by the insertion of the circular cylinder. Most water wave applications are concerned with the influence of the channel walls on the flow: laboratory wave tanks provide a substitute for the open-sea situation, but the presence of the channel walls must be quantified.

An application to water waves was given by Spring and Monkmeyer (1975) using the method of images. The authors stressed the importance of the channel walls by comparing their solutions with those of McCamy and Fuchs (1954), which apply to the open sea situation. One set of experimental data

was presented with disparity evident between the theoretical and experimental results.

This paper considers the problem, from a water wave viewpoint, in two ways. Firstly the image method is reconsidered and secondly a solution in terms of rectangular transmission modes is sought.

One of the difficulties with the image method is the presence of a number of slowly convergent series which must be calculated to determine the matrix coefficients. This difficulty can be circumvented by use of an integral formulation and a semi-recursive algorithm. Transmission and reflection coefficients are also calculated and these are shown to possess some interesting features.

The formulation of the problem in terms of rectangular transmission modes involves the introduction of a matching condition in addition to the existing boundary conditions. A redefinition of the problem is shown to provide an effective mechanism for the removal of the difficulties associated with the matching condition and the solution can then be obtained. The image solution is thought to work best for small values of $^a/b$ whereas the mode solution improves in efficiency $^a/b$ increases; thus the two solutions can be regarded as complementary.

Most of the previous work has been restricted to the case in which the cylinder is at the centre of the channel. Schwinger and Suzuki give approximate solutions for an off-centre cylinder but the 'exact' numerical methods do not work. One advantage of the mode solution is that it can readily be extended to deal with off-centre cylinders and the procedure for doing this will be outlined.

One noticeable feature of this fundamental water wave problem is the lack of good experimental data. Theory will be compared with experiment whenever possible.

References

Marcuwitz, N. 1951 Waveguide Handbook McGraw-Hill (New York).

McCamy, R.C. & Fuchs R.A. 1954 Wave Force on Piles - A Diffraction Theory, Beach Erosion Board Technical Memorandum No. 69.

Nielsen, E.D. 1969 Scattering by a cylindrical post of complex permittivity in a waveguide, I.E.E.E. Trans. M.T.T. 17(3), 148-153.

Schwinger, J. & Saxon D.S. 1968 Discontinuities in waveguides, Gordon and Breach (New York).

Spring, B.W. & Monkmeyer P.L. 1975 Interaction of plane waves with a row of cylinders, Proc. 3rd Speciality Conf. on Civil Eng. in Oceans, A.S.C.E. (Newark, Del.), 979-998.

Suzuki, M. 1955 Dielectric elements in a rectangular waveguide, J. of Inst. of Elec. Comm. Engineers of Japan, 22-28.

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Discussion

Hung: The author is to be congratulated for finding the interesting features of the transmission and reflection coefficients for the free wave modes. An exact solution does exist (1) however, for the case where the cylinder is in an offset position in the channel. In this method, we also provided an alternative and simpler way to ensure the convergence of the slowly converging infinite series.

(1) Eatock Taylor, R. and Hung, S.M. Mean drift forces for an articulated column oscillating in a wave tank. Applied Ocean Research, 1985, pp.66-78.

Thomas: I have read the paper by Eatock Taylor and Hung with regard to the method given there for the summation of the slowly converging infinite series. The method proposed by Eatock Taylor and Hung is approximate because an asymptotic expression is used to approximate the terms in the series.

The method which I have presented in this paper is exact and illustrates clearly the importance of the number of propagating modes on the sum of the series.