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SCATTERING OF SURFACE WAVES OVER A VARIABLE BOTTOM TOPOGRAPHY: "THE EKOFISK EQUATION"

The familiar linearized water wave theory describes water waves by means of a velocity potential in the water volume. The potential, Φ, satisfies the 3D Laplace equation in the volume. Boundary conditions at the bottom and free surface as well as in the far field are also prescribed. This information is, in principle, sufficient to determine the potential everywhere. Once the potential is determined, physical quantities, e.g. pressure and surface elevation, are derived through straight-forward relations.

In general this results in a formidable computational task resulting from the need to solve a full 3D boundary value problem in a large volume.

One strategy to overcome this problem is to work with approximate equations for the surface elevation only. Two examples are the Long Wave Equation and the Mild Slope Equation. They are both 2D and thus reduce the computational effort considerably.

Towards the end of 1984 we were given the task to come up with a scheme to reduce the wave amplitudes at the EKOFISK oil field in the North Sea. The background is the subsidence of the ocean floor which gradually will make the free board to the platform decks dangerously low. Our proposal to achieve the required wave reduction was to place boxes at the ocean floor to create interference minima at the wanted locations. This proposal prompted research into new ways to compute the interference patterns. The water depth at the EKOFISK field is too deep to justify the use of the Long Wave Equation, and the side walls of the boxes are to steep to justify the use of the Mild Slope Equation.

We have therefore deduced a general 2D differential equation for the surface elevation.

The equation is (after transforming to dimensionless coordinates).

$$\sum_{n=0}^{\infty} \frac{(-1)^n h^{2n}}{(2n)!} \left\{ \frac{h}{2n+1} \nabla^{2n+2}\eta + \left(1 - \frac{h}{(2n+1)}\right) \nabla h \, \nabla(\nabla^{2n}\eta) + \nabla^{2n}\eta \right\} = 0$$

We denote this equation by the name "The Ekofisk Equation"

$h = h(x,y)$ - water depth

$\eta = \eta(x,y)$ - amplitude of surface elevation.

The equation is valid and exact as long as the bottom topography is a single valued function of x and y; i.e., the bottom profile is not allowed to have hangover.

This equation looks frightening being of infinite order. It turns out, however, that one can introduce an "ansatz" simplifying matters considerably.

If we put

$$\eta = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(\mu,\nu) e^{i(\mu x + \nu y)} d\mu d\nu$$

the equation transforms into:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} C(\mu,\nu) e^{i(\mu x+\nu y)} \{-\sigma \sinh\sigma h + \cosh\sigma h$$

$$+ i\left(\mu \frac{\partial h}{\partial x} + \nu \frac{\partial h}{\partial y}\right) \left(\cosh\sigma h - \frac{\sinh\sigma h}{\sigma}\right)\} d\mu d\nu = 0.$$

Here $\sigma = \surd(\mu^2+\nu^2)$.

We are now left with the computational task of determining $C(\mu,v)$ such that the above double integral vanishes for all x and y. In addition the conditions at infinity puts other restraints on $C(\mu,v)$.

Although the Ekofisk endavour spurred systematic work on the scattering problem. much of the material presented in this paper dates back some 10 years.