Drift forces on multi-element structures

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Recent work by Eatock-Taylor and Hung [1] concerning mean drift forces on multi-column structures has shown that interaction effects between structural elements may be very important at low frequencies. Their numerical results suggest that for certain geometries the horizontal drift force on a group of N bodies may be of the order of N^2 times the force on a single body when the incident waves are long compared to the overall body size. In the present work this behaviour is investigated analytically. In particular, the long wave limit of the drift force is considered for an array of vertical cylinders extending throughout the water depth and for an array of hemispheres floating on water of infinite depth.

As is well known, the mean horizontal drift force on a body may be calculated from the far field form of the linear diffraction potential. Here the linearised problem of the scattering of long waves by an array of bodies is solved using the method of matched asymptotic expansions under the assumptions that $\mu = kL \ll 1$ and $\varepsilon = a/L \ll 1$, where k is the wavenumber, L is a typical body spacing and a is a typical body radius. In addition, it is assumed that the total horizontal extent of the array is restricted to be much less than the wavelength, thus the present theory does not apply to 'infinite' arrays.

The solution procedure follows that used by Balsa [2,3] in work in acoustics on low frequency flows through arrays of bodies. Three flow regions are distinguished: an outer region at large distances from the array where the length scale is k^{-1}, an intermediate region within the array, (but not 'close' to any body), where the length scale is L and an inner region adjacent to each body where the length scale is a. In the outer region the scattered wave appears to be the result of singularities at a single origin whilst in the intermediate region the disturbance appears to be generated by singularities within each body. The velocity potentials in each of the three regions are expanded in terms of gauge functions in μ and the outer and intermediate expansions matched so that the leading order outer solution is determined in terms of constants in the intermediate region. The potentials in the intermediate and inner region are further expanded in terms of gauge functions in ε. The body boundary conditions are satisfied in the inner regions and matching determines the intermediate solution and hence the outer solution. The final form of the far field potential is a double expansion in μ and ε.

To demonstrate the interaction effects, the ratio $F^{(2)}$ of the drift force in the direction of wave advance on N bodies to the corresponding force on an isolated body is calculated. For an array of identical vertical cylinders

$$\lim_{\mu \to 0} F^{(2)} = N^2 - \frac{6\varepsilon^2}{5} N \sum_{n=1}^{N} \sum_{\substack{p=1 \\ p \neq n}}^{N} \frac{\cos(\beta - 2\alpha_{np})}{(R_{np}/L)^2} + O(\varepsilon^4)$$

where β is the direction of wave advance and (R_{np}, α_{np}) are the (horizontal) polar coordinates of body p relative to body n. Thus the drift force on N cylinders is N^2 times the value for an isolated cylinder only in the limit as ε tends to zero. The existence of the $O(\varepsilon^2)$ term is consistent with the calculations for two cylinders presented by Eatock-Taylor and Hung [1]; for two closely spaced cylinders with the line of centres perpendicular to the direction of wave advance their results show further drift force enhancement over and above the N^2-fold increase. It is interesting to note that the $O(\varepsilon^2)$ term is identically zero for a number of specific geometries. This includes any number, ($\geqslant$ 3), of cylinders at the vertices of a regular polygon, irrespective of the direction of wave advance and a pair of cylinders with their line of centres inclined at an angle of $\pi/4$ to the direction of wave advance. The sign of this term may change with the orientation to the direction of wave advance. For example, for two cylinders aligned with the wave direction,

$$\lim_{\mu \to 0} F^{(2)} = 4(1 - 6\varepsilon^2/5 + O(\varepsilon^4))$$

whilst when their line of centres is perpendicular to the waves,

$$\lim_{\mu \to 0} F^{(2)} = 4(1 + 6\varepsilon^2/5 + O(\varepsilon^4))$$

Thus, interaction effects are stronger in the latter case.

For an array of identical floating hemispheres

$$\lim_{\mu \to 0} F^{(2)} = N^2$$

exactly. The difference in these two results is related to the scattering properties of an individual body. For long waves, the leading order far field scattering potential for a vertical cylinder is a combination of source and dipole terms whereas the corresponding potential for a hemisphere has only source terms. The latter form of the potential will be approximately valid for most floating bodies, provided the water is not too shallow and so an N^2 drift force enhancement is likely with small higher order interaction effects relative to the cylinder case.

References

1. R. Eatock-Taylor and S. M. Hung. Wave drift force enhancement effects in multi-column structures. Applied Ocean Research, 7, 128-137, 1985.
2. T. F. Balsa. Low frequency two dimenional flows through a sparse array of bodies. J. Sound and Vibration, 82, 489-504, 1982.
3. T. F. Balsa. Low frequency flows through an array of airfoils. J. Sound and Vibration, 86, 353-367, 1983.

Discussion

Hung: Dr McIver suggested that given the assumption of body size to separation ratio, the Kochin function $\mathcal{A}(\theta)$ of a group of bodies is the sum of that of individual bodies. Is there any justification for this? I am of the opinion that instead of saying $\mathcal{A}(\theta) \sim N\mathcal{A}_1(\theta)$ we should write $|\mathcal{A}(\theta)| \sim N|\mathcal{A}_1(\theta)|$ noting that $\mathcal{A}(\theta)$ is usually complex.

Reference

R. Eatock Taylor and S.M. Hung, Wave drift enhancement effects for semisubmersible and TLP systems, 1986 OMAE, pp.273-280.

McIver: Certainly for the geometries considered $\mathcal{A}(\theta)$ is pure imaginary to leading order so that it is sufficient to write $\mathcal{A}(\theta) \sim N\mathcal{A}_1(\theta)$. In general, of course, the discusser is quite correct.