Non-Linear Hydrodynamic Forces Acting on Two-Dimensional Bodies Oscillating with a Large Amplitude

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1.Introduction

The approaches in the non-linear problems may be roughly classified into two groups, a perturbation method and a numerical method. On the other hand, there are a few papers regarding inconsistent non-linear theories where the linearized free surface is retained but the body boundary condition is exactly satisfied [1-2]. In such problems, the study of Chapman [1] is especially interesting. He claimed that the problems could be treated numerically by advancing in time as a series of impulse problems with new body shapes at each time step.

The author also suggested that the problems could be formulated by making use of the impulse response function and could be solved by memory effect function which was given from the wave amplitude ratio in the frequency domain [2].

This paper describes a estimating method of the hydrodynamic forces acting on the cylindrical bodies based on the impulse response technique. The procedure of the calculation is based on the theory where the free surface elevation is assumed small. It is also assumed that the interval of impulsive motions are small enough. Numerical calculations are made for a semi-submersible circular cylinder and two semi-submersible elliptic cylinders.

None of the reports clarified the case where the under water form of the body suddenly changes; such a case is observed on semi-submersible offshore structures in transit condition. In the present paper, the problem that the rectangular cylinder is heaving with a large amplitude in the vicinity of the free surface is also dealt.

2.Problem formulation

Following assumptions and conditions are employed in the present study.

(1) The fluid is inviscid, incompressible and irrotational.
(2) The amplitude of the motion of the body is large, and therefore the fluctuation of the wetted part and the effects of displacement are considered. However, the free-surface equations are linearized.
(3) The body is a two dimensional body with arbitrary shape.
(4) The body is at rest at $t<0$.
(5) The water depth is infinite.

In the hydrodynamic boundary-value problem for a two-dimensional body, the formulation in terms of the impulse response function based on the linear potential theory has been introduced by many researchers [3-4]. It seems generally that this method can not be used in the problem that the boundary condition changed every moment. Because the velocity potential by the impulse response function can be represented only for the case where the body is fixed within the limits of the same boundary condition. However, the memory effect function which has an influence upon the next time steps reduces rapidly. Therefore if it can be considered that the past impulsive motion scarcely affects the present motion, we shall be able to use the same method, in principle, even for the case where the body form under the free surface is changing at each time step. We make an assumption; if the interval of the impulsive motion is small enough, the boundary condition on a body surface is satisfied. With this assumption, the velocity potential by the impulse response function can be written as

$$\Phi_I(x,y;t-\tau) = \Psi_{\Omega(\tau)}(x,y)\delta(t-\tau) + \chi_{\Omega(\tau)}(x,y;t-\tau), \qquad (1)$$

where $\delta(t)$: Dirac's delta function

Ψ is the velocity potential during the impulsive motion and X describes the flow due to the existence of the free surface after the impulsive motion. And the suffix $\Omega(\tau)$ means that each function must satisfy the body boundary condition on the underwater form of the body $\Omega(\tau)$.

With the preceding assumptions, the velocity potential of an arbitrary motion with the velocity v(t) can be expressed by Duhamel's formula as follows;

$$\Phi_H(x,y;t) = \int_0^t v(\tau)\Phi_I(t-\tau)d\tau. \qquad (2)$$

By substitution of (1),(2) it follows that

$$\Phi_H(x,y;t) = \Psi_{\Omega(t)}(x,y)v(t) + \int_0^t \chi_{\Omega(t)}(x,y;t-\tau)v(\tau)d\tau. \qquad (3)$$

The linearized hydrodynamic pressure on a body is given by Bernoulli's theorem as follows;

$$P_H(t) = -\frac{\partial \Phi_H}{\partial t}$$

$$= -\rho\dot{\Psi}_{\Omega(t)}v(t) - \rho\Psi_{\Omega(t)}(x,y)\dot{v}(t)$$

$$-\rho\int_0^t \dot{\chi}_{\Omega(t)}(x,y;t-\tau)v(\tau)d\tau, \qquad (4)$$

where ρ is the fluid density.
Then the hydrodynamic force can be written as follows;

$$F_{HH}(t) = -M_{\Omega(t)}(\infty)\dot{v}(t) - m_{\Omega(t)}v(t) - \int_0^t K_{\Omega(t)}(t-\tau)v(\tau)d\tau, \qquad (5)$$

where

$$m_{\Omega(t)} = -\rho\int_{\Omega(t)} \dot{\Psi}_{\Omega(t)}(x,y)\frac{\partial y_{\Omega(t)}}{\partial n_{\Omega(t)}} dS. \qquad (6)$$

$M_{\Omega(t)}(\infty)$ is added mass of the underwater form of the body $\Omega(t)$ with a frequency tending to infinity in the harmonic oscillation. $K_{\Omega(t)}(t)$ is the memory effect function of the wetted part $\Omega(t)$, and it can be written as follows [4];

$$K_{\Omega(t)}(t) = -\rho \int_{\Omega(t)} \dot{\chi}_{\Omega(t)}(x,y;t) \frac{\partial y_{\Omega(t)}}{\partial n_{\Omega(t)}} dS$$

$$= \frac{2}{\pi} \int_0^\infty N_{\Omega(t)}(\omega) \cos \omega t \, d\omega \ . \qquad (7)$$

where $N_{\Omega(t)}(\omega)$ is the frequency-dependent damping coefficient of the wetted part $\Omega(t)$.

By substitution of (5),(7) it follows that

$$F_{HH}(t) = -M_{\Omega(t)}(\infty)\dot{v}(t) - m_{\Omega(t)} v(t)$$

$$= -\frac{2}{\pi} \int_0^\infty N_{\Omega(t)}(\omega) d\omega \int_0^t v(\tau) \cos \omega(t-\tau) \, d\tau \ . \qquad (8)$$

The second term in the right hand side of equation (8) is derived from the fluctuation of the wetted part. It is obvious that this term is a inphase component with the damping, because it is proportional to the velocity.

In order to confirm the validity of the present method, the hydrodynamic force acting on a body which is oscillating periodically is calculated. Let the motion of the body be given as follows;

$$y = y_a \sin \omega_0 t \ , \qquad (9)$$

where ω_0 is circular frequency.
Then we can obtain the following equation from equations (8),(9).

$$F_{HH}(t) = M_{\Omega(t)}(\infty)\omega_0^2 y_a \sin \omega_0 t - m_{\Omega(t)} \omega_0 y_a \cos \omega_0 t$$

$$- \frac{2\omega_0 y_a}{\pi} \int_0^\infty N_{\Omega(t)}(\omega) d\omega \int_0^t \cos \omega_0 \tau \cos \omega(t-\tau) \, d\tau \ .$$

$$(10)$$

By integrating with respect to τ, the required hydrodynamic force can be written as follows;

$$F_{HH}(t) = M_{\Omega(t)}(\infty)\omega_0^2 y_a \sin \omega_0 t - m_{\Omega(t)} \omega_0 y_a \cos \omega_0 t$$

$$- \frac{2\omega_0 y_a}{\pi} \int_0^\infty N_{\Omega(t)}(\omega) \frac{1}{\omega_0^2 - \bar{\omega}^2} [\omega_0 \sin \omega_0 t - \omega \sin \omega t] \, d\omega \ .$$

$$(11)$$

Equation (11) means that the hydrodynamic force on a body oscillating with a large amplitude in the time domain can be expressed by making use of the damping coefficient in the frequency domain.

References

1) Chapman,R.B. : Large amplitude transient motion of two-dimensional floating bodies, J.Ship Res., Vol.23, No.1, March 1974.
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3) Oortmerssen,G.Van : The Motions of a Moored Ship in Waves, Publication No.510, Netherland Ship Model Basin, 1976.
4) Takagi,M.et al. : Comparisons of Simulation Methods for Motions of a Moored Body in Waves, Proc. of 3rd Int. Offshore Mechanics and Arctic Eng., ASME, 1984.

Discussion

Kim: Your method satisfies the body boundary condition exactly, while maintaining the linear free-surface condition. For the rectangular geometry you choose, strong nonlinearity of the free-surface is expected; this may be one of the reasons theory and experiment do not agree well. If we consider the resonance of the geometry whose waterplane area is small, a large amplitude motion is expected without generating high amplitude waves. I think this is the ideal case to which your theory can be applied.

Higo: My presentation consists of two parts. The former part is about the theory by making use of the impulse response technique. And in order to confirm the validity of this method, the calculated results are compared with the experimental one of the hydrodynamic force acting on the circular cylinder and the elliptic cylinders heaving with a large amplitude. So it is observed that the calculated values show good agreement with the experiments under the floating condition.

The latter is the case of the experiments about the rectangular cylinder. In this case, it is considered that the nonlinearity of the free surface condition is strong. So I think that the application of the above-mentioned method would be inappropriate. But I had practical needs to analyse this problem for semi-submersible offshore structures in the transit condition. So I chose to apply this method to get some information on this phenomenon.