THE MAXIMUM EFFICIENCY OF WAVE-ENERGY DEVICES NEAR COAST LINES

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It is well known (Evans (1980)) that the maximum mean power that N wave-energy devices, each operating in a single degree of freedom, can absorb is

$$P_{max} = \tfrac{1}{8}X^*B^{-1}X$$

where $$B = \{B_{mn}\} \quad m,n = 1,2,\ldots N \qquad (m = 1,2,\ldots N)$$

is the radiation damping matrix for the group of devices, and $X = \{X_m\}$ is the time-independent vector of exciting forces on each device in the direction of its subsequent motion, due to the long-crested incident wave of radian frequency ω.

Now it can be shown (Newman (1976)) that

$$B_{mn} = \frac{1}{8\lambda P_w}\int_0^{2\pi} X_m(\theta)\overline{X}_n(\theta)\,d\theta \tag{1}$$

where λ is the incident wave length and P_w is the mean power per unit crest length of the incident wave. Here θ is the angle of incidence of the incident wave. For an array of 'point absorbers' we may approximate $X_m(\theta)$ by the value of the incident wave potential at the mth absorber.

Thus for a line of N point absorbers equally spaced a distance d apart, making an angle β to the direction of the incident wave, it turns out that the maximum capture width ℓ_{max} is

$$k\ell_{max} = k\,\frac{P_{max}}{P_w} = L^*J^{-1}L$$

where $$J_{mn} = J_0\{kd(m-n)\}$$

and $$L_m = \exp\{ikdm\sin\beta\}, \quad k = 2\pi/\lambda$$

In particular if $N = 1$ we get the well-known result $\ell_{max} = \lambda/2\pi$.

It is possible to generalise these results to allow for the presence of a coastline thus enabling estimates to be made of the maximum capture width of one or more devices of the type currently being considered by Kvaerner-Brug A/S off Bergen in Norway. To fix ideas we consider an infinitely long vertical coastline making an angle β to the incident waves. Then it turns out that the reflected wave makes no contribution to the stationary-phase arguments involved in re-deriving a formula corresponding to (1) and appropriate to the presence of the coastline; the only changes needed are that the integration is now taken from zero to π in (1), and the appropriate $X(\theta)$ including the reflected wave must be used.

Thus we find that for an isolated point absorber

$$\ell_{max} = \lambda|\phi(\beta)|^2 \Big/ \int_0^{\pi} |\phi(\theta)|^2 d\theta \qquad (2)$$

where $\phi(\beta)$ is the incident plus scattered potential at the absorber due to waves approaching from an angle β. Notice that when dealing with point absorbers it is sufficient to use the incident potential rather than the exciting force at the point in question.

A simple example is given by a single point absorber a distance b from a perfectly reflecting coastline, when we find that

$$\ell_{max}(kb,\beta) = \frac{\lambda}{\pi} \frac{(1 + \cos(2kb \sin\beta))}{(1 + J_0(kb))} \qquad (3)$$

reducing to $\ell_{max} = \frac{\lambda}{\pi}$ when the device is embedded in the coastline.

These results can be generalised to any number of devices and to include a coastline which is not perfectly reflecting but which absorbs a proportion of the energy incident upon it.

If we impose the impedance condition

$$\phi_y = ikp^{-1}\phi \quad \text{on} \quad y = 0,$$

we can model a partly reflecting coastline with a reflection coefficient $R = -(1 - p\sin\beta)/(1 + p\sin\beta)$ which for $p > 0$ models a loss of energy at the coastline.

Thus, for example, the maximum capture width for a single device embedded in an absorbing coastline of this type turns out to be

$$\ell_{max}(p,\beta) = \frac{\lambda \sin^2\beta}{(1 + p \sin\beta)^2} \Big/ \int_0^{\pi} \frac{\sin^2 t}{(1 + p \sin t)^2} dt \qquad (4)$$

again reducing to known results when $p = \infty$ corresponding to a totally reflecting coastline.

The general expression for an absorber a distance b from the coastline is

$$\ell_{max}(p,\beta,kb) = \lambda I(p,\beta,kb) \Big/ \int_0^{\pi} I(p,\theta,kb) d\theta$$

where $$I(p,\beta,kb) = \frac{p^2 \sin^2\beta \cos^2(kb\sin\beta) + 4\sin^2(kb\sin\beta)}{(1 + p \sin\beta)^2} \qquad (5)$$

Recent proposals in the UK envisage the exploitation of naturally occurring inlets off small islands as possible wave-power sites. To estimate the maximum absorption width of such inlets, regarded as point absorbers, it is sufficient to determine the incident plus scattered field at the inlet due to the presence of the island, and then use (2) with π replaced by 2π.

A simple illustration is provided by an inlet or point absorber on a circular island. The incident plus scattered field for a wave incident upon a circular island of radius a is easily determined and we find that

$$\ell_{max} = \frac{\lambda}{2\pi} |S|^2/T \qquad (6)$$

where

$$S = \sum_{n=0}^{\infty} \frac{\varepsilon_n (+i)^n \cos n\beta}{H_n'(ka)}$$

$$T = \sum_{n=0}^{\infty} \varepsilon_n \Big/ |H_n'(ka)|^2 \qquad \varepsilon_n = 2 \quad n \neq 0, \quad \varepsilon_0 = 1$$

Results

Fig. 1 shows the results of computations based on (3) for different angles of incidence of the incoming wave. It is clear that the presence of the reflecting coastline can considerably enhance the maximum capture width for certain values of b/λ.

In Fig. 2 (4) has been computed to explore the effect of varying amounts of reflection at the coastline on a small Kvaerner type device. It can be seen that for angles in excess of about 45° the capture width ratio is actually increased for p finite.

Fig. 3, derived from (6), shows the effect of a point absorber on a circular island. It can be seen from Fig. 3(a) that the capture width ratio lies between the two limiting results for $ka = 0,\infty$ as expected. Fig. 3(b) shows the effect of varying the position of the absorber on the island, relative to the incident wave. The results are as expected with the least power absorbed when the device is in the lee of the waves.

References

1. Evans, D.V. Some analytic results for two and three dimensional wave energy absorbers, in Power from Sea Waves ed. B. Count, London (N.Y., Academic) (1980) 213-250.

2. Newman, J.N. The interaction of stationary vessels with regular waves. Proc. 11th Symp. Naval Hydrodynamics, London (1976) 491-501.

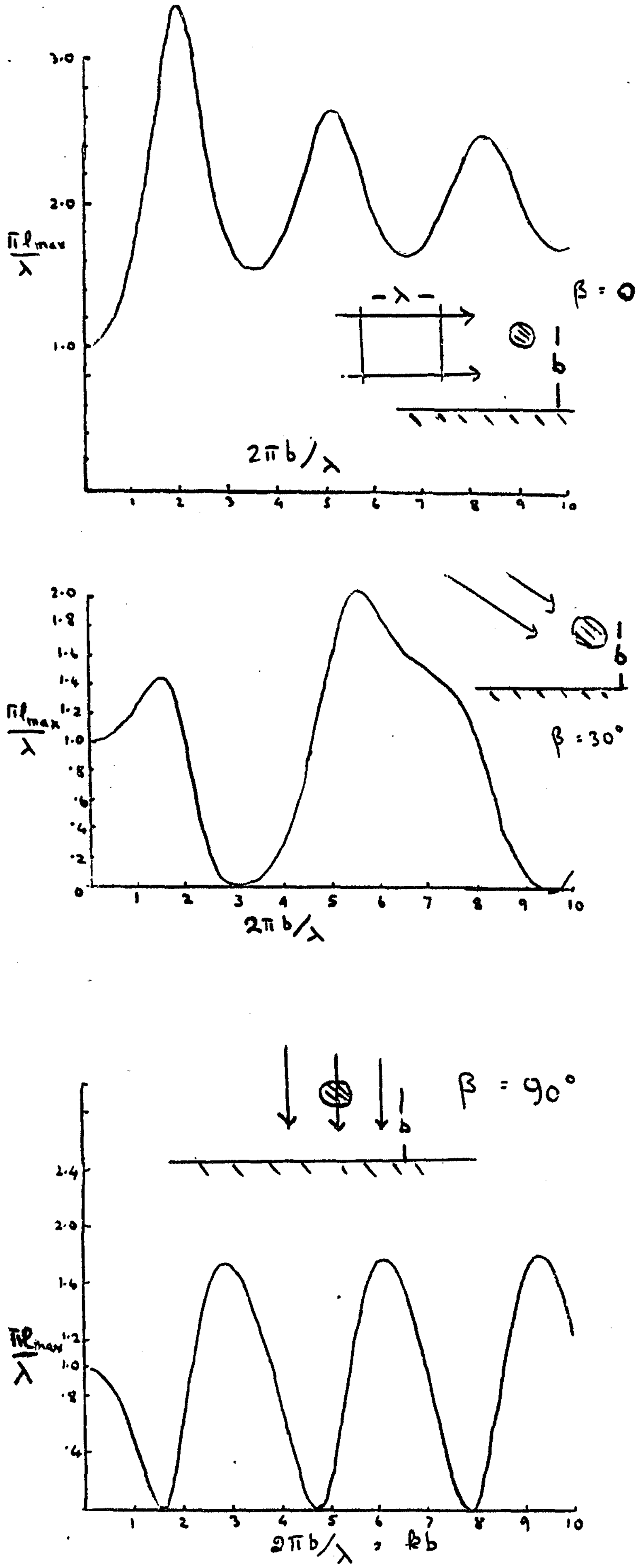


Fig. 1.

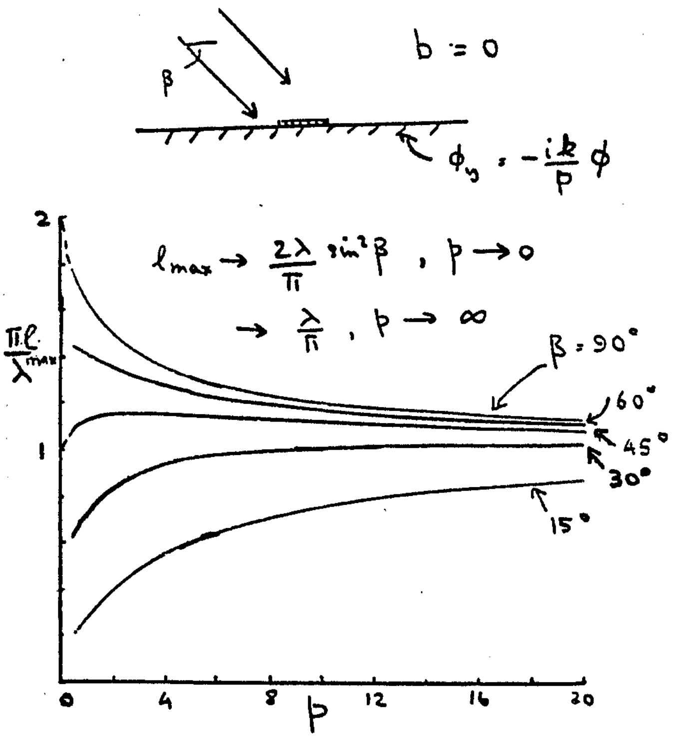


Fig. 2

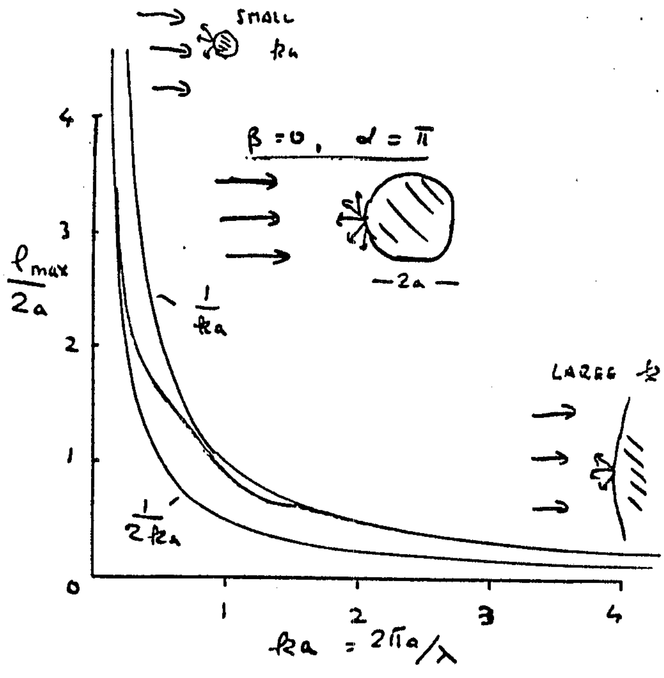
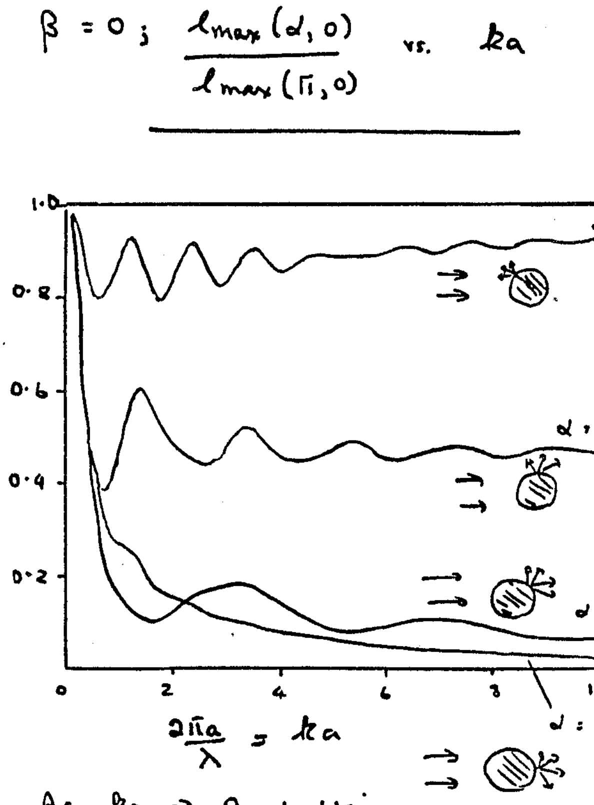


Fig. 3 (a)



As ka → 0 position of absorber is irrelevant.

Fig 3 (b)

Discussion

X.J. Wu: We have conducted some research projects for industry towards utilisation of wave power for the ocean buoy. It was found that the real sea wave power absorption efficiency was rather low though the device might be designed in a better version based on calculations and various tests for each individual component of the whole device. In particular, my student did computation and gave an average output of 2.91w for an air turbine wave power buoy in a sea site near Shanghai. Fortunately, the averaged value of 100 days real sea test record provided by industry was 2.85w. The real sea record of the generated electric current appeared changeable and erratic and therefore no stable power output could be obtained (Wu et al, 6th OMAE, 1987).