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Extended abstract of presentation by E.O.Tuck of paper :

Waves generated by air flow from a stationary hovercraft

by

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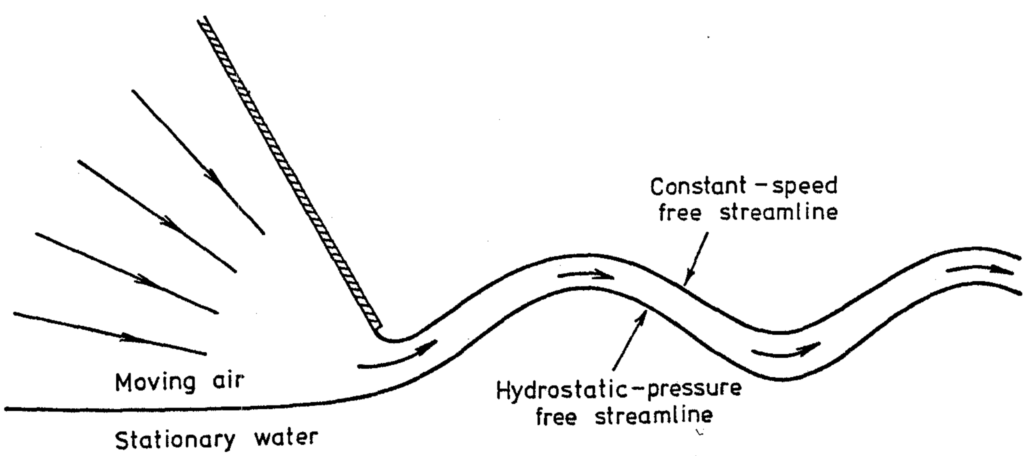

If air is caused to move in a non-uniform manner above a water surface, that surface will deform under the action of the non-uniform aerodynamic pressure, whether or not the water moves. This talk concerns only those cases where the water remains at rest, or in which its motion can be neglected relative to that of the air. In such cases, the equilibrium free-surface configuration represents a balance between aerodynamic and hydrostatic pressures.

The boundary condition on the interface between air and water can be shown (Tuck, 1975) to be equivalent to that for moving water with a constant-pressure free surface. Some well known water-wave solutions (e.g. deep-water Stokes waves, as computed by Schwartz, 1974) can simply be turned upside down and re-interpreted as relevant to the present context. However, an interesting class of problems that does not have a direct water-wave analogue is that in which the air layer is of finite thickness, its upper boundary being a constant-speed free streamline.

An example of such a flow is a vertically downward jet of air impinging upon the water surface, where it splits symmetrically and then spreads out over the surface, generating waves in the process. This problem was studied first by Olmstead and Raynor (1964), and more recently by Vanden Broeck (1981). It is the purpose of the present paper to study another (unsymmetric) example of a generating mechanism, as well as to investigate in their own right the non-linear waves that are generated.

As an idealised model of the edge-seal zone of a hovercraft (see e.g. Trillo 1971), or the trailing edge of an airfoil flying close above water (Tuck 1985, Grundy 1986), consider the steady flow sketched in Figure 1. Air (assumed incompressible) is caused to flow inward from infinity on the left, in a sink-like manner, in a sector region bounded by a plane wall and a static water surface. The air then passes through a gap beneath the lower edge of the wall and the water, and emerges on the right as a jet, that flows to infinity in a layer above the water surface. At a great distance to the right, there appears an asymptotically-periodic wave, and one of our tasks is to compute the amplitude of the wave generated, as a function of forcing parameters such as the net volume flux of air and the gap width.

The wave is small only when the air flux is small, or, more precisely, when a suitable Froude number based on this flux is small. In the limit when this Froude number tends to zero, the water surface appears as a rigid wall, and the air jet flows over that wall in a non-wave-like manner. This is a standard free-streamline problem, and the solution can be obtained easily in closed form by hodograph methods.

A generalisation to finite Froude number allows the water surface to deform, and this problem can be converted to a non-linear integral equation for the hodograph variables (velocity magnitude and direction) along the air-water free boundary. This integral equation is then solved numerically for various values of the Froude number and wall angle.

As the Froude number increases from zero (in effect, as we blow harder relative to the available gap), the violence of the disturbance to the water surface increases, and the amplitude of the far-field waves increases. However, at least in theory, extremely steep waves can be generated, with amplitudes many times the gap width or the air-layer thickness. In practice, such large free-surface deformations would suggest a break up in the form of spray.

In order to throw more light on these large-amplitude waves, an independent study is provided of the far-field limit, i.e. of purely periodic plane waves on an interface between dynamic air and static water. For this purpose, a simple Fourier series representation of the solution is truncated to a finite number of terms, the coefficients of which are then found by collocation of the boundary condition.

In the limit where the air layer's thickness is far in excess of the wavelength, these waves are exactly the same as deep-water plane progressive (Stokes) waves, turned upside down. Such a "deep-layer" wave has the usual maximum steepness (trough-to-crest height/wavelength) of about 0.141, and is limited by development of a stagnation point in a 120° angle, but at the trough rather than the crest. However, finite air-layer thickness has the opposite effect to that of finite water depth on Stokes waves, in that it allows the steepness to increase above 0.141. For large-to-moderate layer thicknesses, the limiting wave of maximum steepness still has a stagnation point at the trough. However, for moderate-to-small layer thicknesses, the waves become so steep (amplitudes comparable to wavelength) that they are no longer so limited, but continue to steepen until the upper air surface self-intersects.

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Discussion

Schwartz: Can you relate this problem to the conventional Stokes wave problem? For Stokes waves, one can obtain a power series relating the phase speed to the wave amplitude.

Tuck: This problem is conjugate to the Stokes wave problem in water of finite depth.

Wu: You have a momentum source, so the relationship mentioned by Schwartz probably does not exist.

Wehausen: Can you do experiments with some liquid other than air which has a different density from water?

Tuck: I agree that it is easier to do the problem especially if the density ratio is close to unity.

For the present case with an air-water interface the air will blow off in reality. Solutions are computed that possibly could exist, but for high speeds they are probably unstable.

Newman: The most noticeable feature of a hovercraft is the great volume of spray they generate, which leads me to wonder about the stability of the free surface. Could you draw a plot of the pressure distribution above the free surface, and on the same absissa the free surface shape? This would indicate whether the flow is unstable.

Tuck: The wave will become unstable when its amplitude gets large. One could study at what steepness this will occur.

X.J. Wu: Have you considered incident waves and a hovercraft's motions in this study? There is a longitudinal variation in the pressure distribution under the skirts of a hovercraft which should be considered. And the heave motion of the hovercraft also induces further fluctuation of the air pressure.

Tuck: I did not study the hovercraft problem in waves.

Stiassnie: You are neglecting the flow of water below the free-surface and the flow of air above it. In the zero Froude number case the flow resembles one half of a jet, so would it not entrain air from above the "separating" streamline?

Tuck: I agree that entrainment will occur. The problem is subjected to all the difficulties you mentioned even at zero Froude number, also to the problem of spray formation at finite Froude number.

Schwartz: I am thinking again of:

(i) the relationship to Stokes waves. For gravity waves we need to consider the singularity in order to determine the limiting form. For pure capillary waves on the other hand there is a closed-form solution with no singularity. Is it possible to relate your limiting form to the capillary-wave solution?

(ii) Can you pose the problem without gravity?

Tuck: (i) I do not know the answer to that. For large Froude numbers the limitation is similar to that for capillary waves. The waves are only limited by topology. The solution moves smoothly from this limiting case to the more usual Stokes-wave type limitation with a 120 degree stagnation point on the free surface as the Froude number tends to zero.

(ii) It is not possible to do the problem without gravity.

Yeung: I worked on air-cushion vehicle hydrodynamics at the beginning of my career. As a function of Froude number, the wave-resistance coefficient of a two-dimensional constant pressure patch is highly oscillatory with non-decreasing amplitude in the low Froude-number limit. It is not really infinite as you stated. The effect of tapering the edges of the pressure band is to decrease the amplitude of oscillation at low Froude numbers. Perhaps your present model will provide some guidance on the magnitude of the taper.