"A Theoretical Investigation of Wall Reflections" A. Troesch, Univ. of Mich.

Introduction

There are unique problems when considering motion tests of models with zero or near zero speed in experimental basins of finite dimensions. In addition to the incident waves generated by the facility's wave making device, the model itself produced a wave pattern which travels away from the model toward the tank boundary. The tank walls reflect two separate wave systems: incident waves scattered or diffracted by the model, and radiated waves generated by the motions of the model. It is well known that if the radiated waves are at a frequency corresponding to a transverse tank slosh mode, the resonant behavior can make accurate analysis of the measurements virtually impossible. Even at frequencies not near a tank resonant mode, it has been observed that the reflections from the walls still influence the model test results.

The purpose of this work is to get a better understanding of wall reflections. The theoretical model will consider two aspects of the problem: the length of time it takes for body motions to achieve periodic motion when being excited by a periodic force and the influence of the walls on the prediction of the vessel Response Amplitude Operator (RAO) in unbounded waters. The first deals with the time domain description of the motion while the second with the frequency domain.

Time Domain Aspects of Wall Reflection

Preliminary model experiments have shown that the forced heave radiated wave generated by a spherical model in the towing tank is a complicated function of time. That is, when the wave elevation at some distance from the sphere is represented by an amplitude mutiplied by a harmonic function of time, the amplitude does not reach a constant value before reflections from the tank ends arrive at the wave probe. The wave frequency in the experiment did not correspond to a transverse slosh mode and the diameter of the sphere was relatively small compared to the tank width, a ratio of approximately 1:13. A sphere was selected for this test since, in a far-field sense, it has similar characteristics to a large class of offshore structures.

To analytically simulate the transient model test results, time dependent velocity potentials were constructed using the Impulse Response Function (IRF) technique. Following Cummins (1962) and many others, the IRF was found by Fourier transforming the frequency response function, the latter which was determined for harmonic motions. In this particular problem, the quantity of interest is the wave elevation in the center of the tank, at a distance of one-half tank width from the heaving sphere. The solution was constructed using the methods of images where the vertical and parallel walls were replaced by an infinite number of bodies in unbounded water. A far-field approximation in the modeling of the problem is an array of point sources. The IRF for a point source is the classical Cauchy-Poisson problem and the numerical accuracy of the technique was verified through comparison with the analytical results given in Lamb (1945). Since the solution is singular for large time or small radial distance, the potential was evaluated slightly below the free surface.

The IRF for the source and its images are shown in Figures 1 and 2. In Figure 1, the top line is the IRF for the center source evaluated at a point 13 radii away from the origin of the sphere. The next five lines represent the images for the same point. The tank width is 6.7m, so lines 2-6 represent the images at 6.7, 13.4, 20.1, 26.8, and 33.5m respectively. One interesting fact is that the images from 33.5m still contribute significantly to the IRF. This indicates that an 33.5m wide experimental test basin could have reflection problems under certain conditions. Figure 2 shows the sums of the individual IRF's shown in Figure 1. The top line is just the IRF for a single source, line two the sum of a single source and its first set of images, and so on.

To find the wave elevation due to a sphere oscillationg sinusoidally from rest, the IRF as given in the last line of Figure 2 was convolved with a sine wave. Three different frequencies were tried and the theoretical results are shown in Figure 3. The periods of oscillation were 2.03, 1.44, and 0.84 seconds respectively. Experimental results for the same three frequencies are shown in Figure 4. A comparison between experiment and theory indicates that the theoretical modeling technique is reasonable.

Frequency Domain Aspects of Wall Reflection

The goal of many model experiments is to determine the RAO of a vessel in unbounded water. During a typical test, sinusoidal waves are used to excite the body. After the transients have diminished, the wave height and model response are measured to form the RAO as a function of wave encounter frequency.

The influence of the tank walls can be estimated by considering a sphere oscillating between two vertical walls. The sphere is chosen to reduce computational difficulties. The velocity potential can be represented by using the method of images and multipole expansions. Generalizations of Thorne's (1953) multipole expansions using associated Lengendre functions have been given by many, including Barakat (1962), Greenhow (1980), and Hulme (1982).

For a single sphere in unbounded water symmetry arguments relating to the body boundary conditions and body geometry can be used to simplify the evaluation of the hydrodynamic forces. However, if there is more than one sphere present, there are interactions between the spheres which modify this simple symmetry and other methods are required. Greenhow (1980) considers the "nearest neighbor" interactions and Simon (1982) considers the "plane wave" approximation, both which require the sphere radii to be small compared to their separation distance. An alternative is to employ higher order multipoles and higher order wave-free singularities. In this way it is possible to satisfy the body boundary conditions on all the spheres at the same time, including interaction effects.

The actual form of the solution is to include all orders and degrees of wave-type multipoles and wave-free singularities. The source strengths for the center sphere and its images are equal. The source strengths were found by solving a set of matrix equations in the least squares sense. A numerical test was conducted to determine the rate of convergence for the multipole and singularity expansions. Generally convergence was reached after three orders of multipoles and four orders of wave-free singularities were used. This was verified by comparing results with fifth order multipole, sixth order wave-free singularity computations. A comparison with the results given by Greenhow (1980) was made for the heave added mass and damping of two spheres separated by three, five, and ten radii. Figure 5 shows some of the results. The calculations of this work seem to show the same general trends except for the case of the added mass for spheres at a separation of ten radii. Perhaps some of the differences may be due to the "nearest neighbor" assumption.

The computer program that calculates the potential was generalized to accept any number of spheres or ellipsoids from one to one hundred. In the physical world, viscosity will limit the effects of the images far removed from the origin, so it seems reasonable to restrict their number in the theory. Various examples were run to determine the effects of spacing and number of images. The added mass and damping coefficients are shown in Figures 6 for one of the cases. The tank width and number of images are noted on the figure. The curves tend to oscillate about the values for the single sphere in unbounded water which may identify some of the sources of experimental "scatter" in model tests.

References

Barakat, R. (1962). "Vertical Motions of a Floating Sphere in a Sine-Wave Sea". JFM, vol. 13

Cummins, W. (1962). "The Impulse Response Function and Ship Motions". Schiffstechnik Bd 9, Heff 47.

Greenhow, M. (1980). "The Hydrodynamic Interactions of Spherical Wave Power Devices in Surface Wave". Power from Sea Waves. B. Count, ed. Academic Press.

Hulme, A. (1982). "The Wave Forces Acting on a Floating Hemisphere Under-going Forced Periodic Oscillations". JFM, vol 121.

Lamb, H. (1945). Hydrodynamics, 6th edition, Dover, New York.

Simon, M. (1982). "Multiple Scattering in Arrays of Axisymmetric Wave Energy Devices. Part I" JFM, vol 120.

Thorne, R. (1953). "Multipole Expansions in the Theory of Surface Waves". Proc. Camb. Philo. Soc. vol 49.

Acknowledgements: A large part of the work described here will appear in Stuart Cohen's Ph. D. Thesis, The University of Michigan. The project was funded by both the Michigan Sea Grant Program and the Dept. of NAME, U of M.

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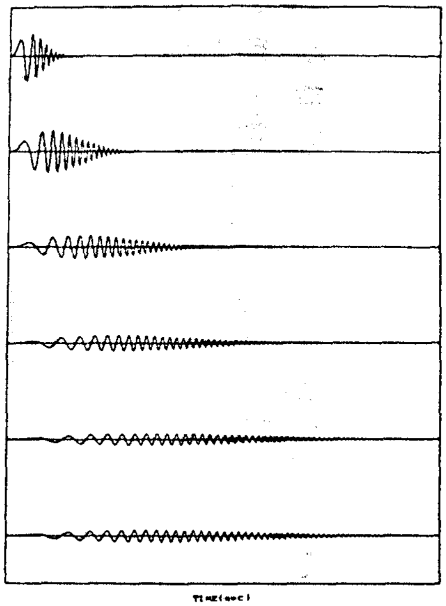

Figure 1

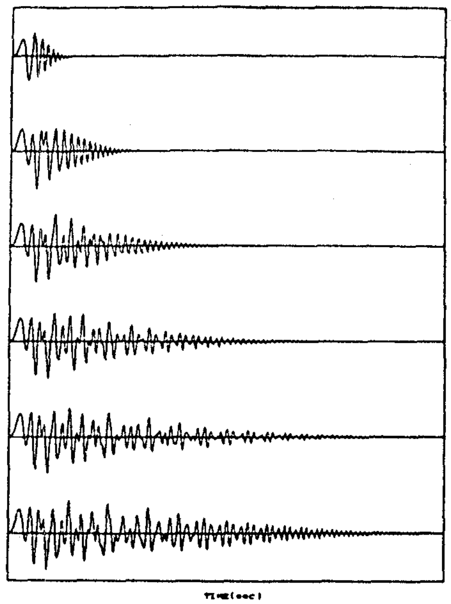

Figure 2

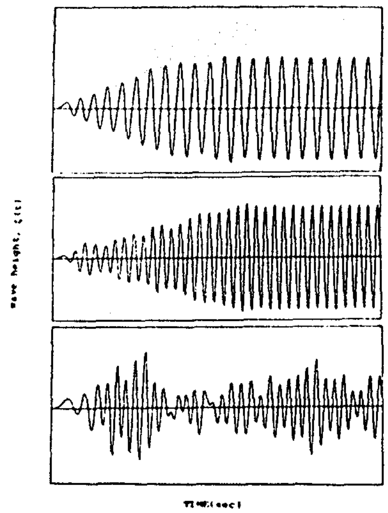

Figure 3

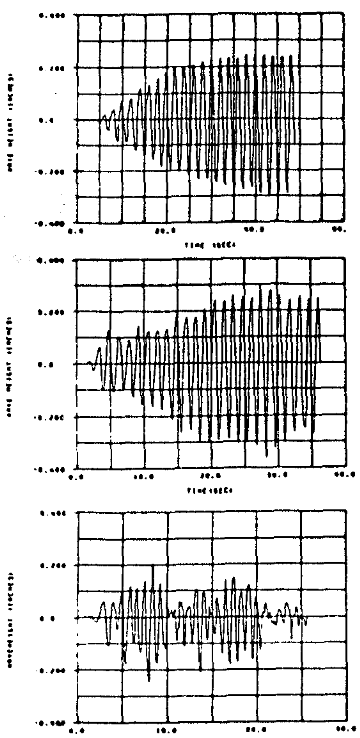

Figure 4

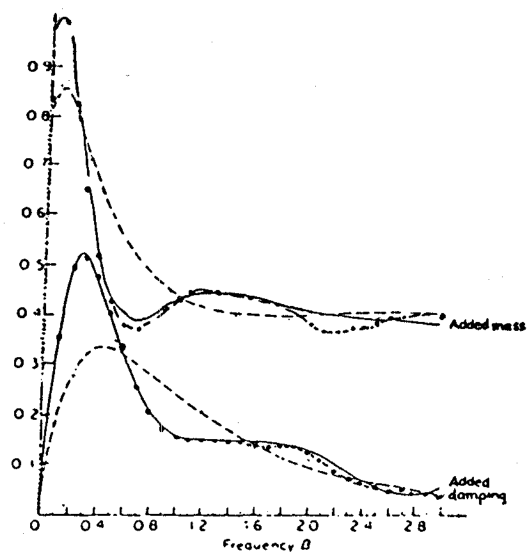

Figure 5

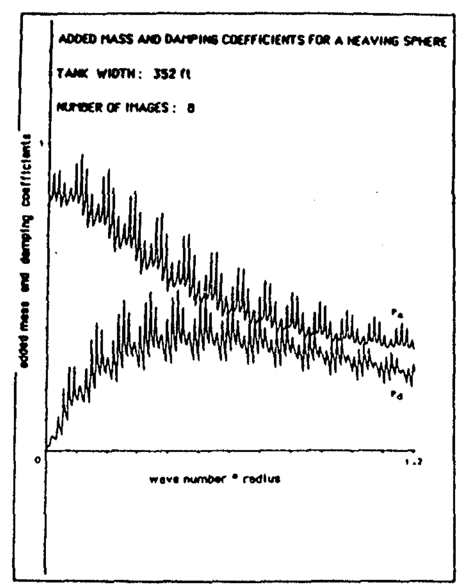

Figure 6

Discussion

Evans: For a body oscillating in a narrow wave tank there is a sequence of frequencies - the cut-off frequencies - at which propagating waves are proscribed. Can this ever be modelled by a finite set of images about the tank walls? Presumably the behavior at high frequencies needs to be modelled correctly to ensure the correct behavior for small time in the time domain.

Troesch: From the time domain point of view, a distant source will not affect the experiment if it is of finite duration.

Yeung: Concerning the points Evans raised, I suggest looking at the results of Figure 2 in the abstract. It appears that the small-time behavior is not affected by an increase in the number of images. However, Evans' argument is that cut-off frequencies in a channel can be modelled only by an infinite summation. Since high-frequency behavior affects small-time response, how does one resolve this apparent contradiction?

Troesch: It is true that truncation reduces small-time response. However, "small time" response refers to the ability of the potential to change rapidly over small time changes, i.e., large values of the time derivative of the potential. This is not the same as an expression for the potential valid for small absolute time. A finite number of images will never yield an infinite response at the sloshing frequencies. But as seen by the graphs of added mass and damping for a finite number of multiple bodies, we can get very large responses. Viscosity and finite experimental run time seem to justify image truncation.

Tuck: For a point slightly below the surface maybe we do see what has been shown. But what about at the free surface?

Troesch: We did try to use a point on the free surface and found reasonable comparison with the classical Cauchy-Poisson problem.

Mehlum: Is dissipation (friction etc.) a reason why truncation of the Bessel series is justified?

Troesch: Yes, viscosity would have that effect.

Evans: Have you looked at a closed form solution for the wave free potential?

Troesch: We tried that and were not successful. For the complete problem. including images, you need more singularities evaluated at various (r, θ, ϕ).
The problem becomes particularly difficult when trying to Fourier transform the closed form solution to get time-domain representations.

Stiassnie: Did you try wave absorbers on the wall?

Troesch: No, our experiences have been that beaches are less than 100% absorbing, and with whatever energy you get back there is significant error.