ON THE NONLINEAR WAVE FORCES ACTING ON PARTIALLY OR FULLY SUBMERGED CYLINDERS IN WAVES

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SUMMARY

The paper deals with the evaluation of the nonlinear hydrodynamic effects arising at oscillations of two-dimensional, arbitrarily shaped cylinders floating in or being fully submerged under the free-surface and in response to a regular wave train. The employed second-order potential theory is based on a regular perturbation expansion in terms of small parameters and Green's function Integral-Equation-Method. The resulting set of Fredholm Integral Equations, to be solved for the first-and second-order potentials, is formulated as a Combined Helmholtz Integral Equation which is free of irregularities. It employs pulsating dipoles and sources along the wetted body surface, as well as at the body's origin, and sources along the free-surface at rest, in case of second-order.

In the present study special attention is paid to the solution of the second-order diffraction problem and the calculation of the resulting second-order wave forces acting on partially or fully submerged cylinders near under the free-surface. Recent experimental results by Kyozuka and Inoue |1| and their comparisons with a similar theory indicated a strong disagreement between theory and experiments especially for cylinders close to the free surface. However, in the present paper, it is shown that the employed second-order theory and numerical procedure predict quite satisfactory the measured first-and second-order wave forces, even for small depths of submergence, thus they prove to be a valuable tool towards better understanding of various nonlinear effects in ship motions.

STATEMENT OF THE PROBLEM

The mathematical modelling of the hydrodynamics of a cylinder, oscillating with finite amplitude in response to a regular, possibly steep wave, has been described in earlier publications |2|. Herein, the second-order diffraction problem for partially or fully submerged cylinders will be focussed.

Perturbation Expansion

The velocity potential describing the fluid motion resulting from the diffraction of a regular wave train of frequency ω due to the presence of a cylinder can be expressed as power series in the small parameters ε_0 and ε_7 in the sense of a regular perturbation expansion:

$$\Phi(x,y;t;\varepsilon_k) = \sum_k \varepsilon_k \varphi_k^{(1)}(x,y)e^{-j\omega t} + \sum_{i,k} \varepsilon_i \varepsilon_k \varphi_{ik}^{(2)}(x,y)e^{-2j\omega t} + O(\varepsilon_k^3) \tag{1}$$

with $i,k=0,7$. In (1) ε_0 represents the wave steepness and ε_7 the ratio of the incident wave amplitude to the maximum half-beam of the body. Furthermore, index 0 pertains for wave and 7 for diffraction.

Rearranging the incident wave (index 0), diffraction (index 7) and interaction potentials (index 07) and considering, that $\varepsilon_0 = \varepsilon_7 \cdot (kb)$, we obtain the combined wave-diffraction potential (index 8):

$$\varphi_8^{(1)} = \varphi_7^{(1)} + (kb)\,\varphi_0^{(1)} \quad , \qquad \varphi_8^{(2)} = \varphi_7^{(2)} + (kb)\,\varphi_{07}^{(2)} \quad , \tag{2}$$

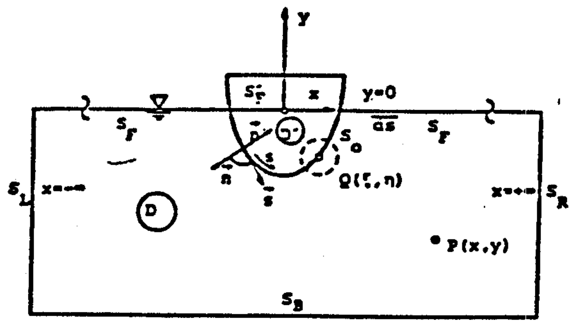
where k represents the wave number and b the maximum half-beam of the body. In (2) the second-order wave potential $\varphi_0^{(2)}$ has been omitted as being trivial in deep water.

The wave-diffraction potentials $\varphi_8^{(1)}$ resp. $\varphi_8^{(2)}$ can now be expressed as the sum of a symmetrical (index 10) and an antisymmetrical (index 9) part, taken with respect to the vertical symmetry axis of the body:

$$\varphi_8^{(1)} = \varphi_9^{(1)} + \varphi_{10}^{(1)} \qquad \varphi_8^{(2)} = \varphi_9^{(2)} + \varphi_{10}^{(2)} \tag{3}$$

Boundary Value Problems

The first-order potentials $\varphi_9^{(1)}$ and $\varphi_{10}^{(1)}$ can be assumed to be known. The second-order potentials $\varphi_9^{(2)}$ and $\varphi_{10}^{(2)}$ are given through the solution of the following uniform boundary-value-problem for $\varphi_i^{(2)}$ (i=9,10)(see sketch for the geometry of the BVP) :



$$\Delta\varphi_i^{(2)} = 0 \qquad (x,y)\,\varepsilon\, D$$

$$\{\frac{\partial}{\partial y} - 4\nu\}\,\varphi_i^{(2)} = L_i^{(2)} \qquad (x,y)\,\varepsilon\, S_F$$

$$\frac{\partial\varphi_i^{(2)}}{\partial n} = 0 \qquad (x,y)\,\varepsilon\, S_0 \tag{4}$$

$$\frac{\partial\varphi_i^{(2)}}{\partial y} = 0 \qquad (x,y)\,\varepsilon\, S_B$$

$$\{\frac{\partial}{\partial x} \mp j4k\}\,\varphi_i^{(2)} = 0 \qquad (x,y)\,\varepsilon\, {}^{S_R}_{S_L}$$

where

$$L_9^{(2)} = j\frac{\omega}{2g}\{4\varphi_{10x}^{(1)}\varphi_{9x}^{(1)} + 6\nu^2\varphi_{10}^{(1)}\varphi_9^{(1)} + \varphi_{10}^{(1)}\varphi_{9xx}^{(1)} + \varphi_9^{(1)}\varphi_{10xx}^{(1)}\} \tag{5.1}$$

$$L_{10}^{(2)} = j\frac{\omega}{2g}\{2(\varphi_{10x}^{(1)2} + \varphi_{9x}^{(1)2}) + 3\nu^2(\varphi_{10}^{(1)2} + \varphi_9^{(1)2}) + \varphi_{10}^{(1)}\varphi_{10xx}^{(1)} + \varphi_9^{(1)}\varphi_{9xx}^{(1)}\} \tag{5.2}$$

Herein $\nu = \omega^2/g$ means the frequency number and the index x a differentiation in the horizontal x direction.

Integral Equation Method

Applying Green's third theorem to the uniform BVP(4) the following Fredholm Integral Equation of second kind for $\varphi_i^{(2)}$ (i = 9,10), taken on the body surface S_0, is obtained:

$$-\pi\varphi_i^{(2)}(P) + \int_{S_0}\varphi_i^{(2)}(Q)(\vec{n}\vec{\nabla})_Q\, G^{(2)}(P,Q)\,ds_Q = I_{F_i}^{(2)}(P)\,,\ \text{for } P\,\varepsilon\, S_0 \tag{6}$$

where the RHS $I_{F_i}^{(2)}$ is given by a Line-Integral over the free-surface S_F, namely:

$$I_{F_i}^{(2)}(P) = -\int_{S_F} L_i^{(2)}(Q)\, G^{(2)}(P,Q)\,ds_Q \tag{7}$$

Green's Function $G^{(2)}$ (P,Q), with P(x,y) being the field and $Q(\xi,\eta)$ the source point, satisfying the adjoint, homogeneous BVP(4), is given in |2| and will be omitted herein. However, in modification to |2|, for avoiding the well known irregular frequencies the conventional Green's Function is modified on the basis of the work by Ogilvie - Shin |3|. Herein, this idea is extended to integral equations of Helmholtz type, like (6), being characterized by pulsating normal dipoles along S_0 and pertaining for the potential itself instead of for the source strength. Further the method has been applied to both the first-and second-order subproblems for $\varphi_i^{(1)}$ and $\varphi_i^{(2)}$. The modified Green's function used is:

$$\tilde{G}^{(2)}(P,Q) = G^{(2)}(P,Q) + C_0\{C_1 G_Q^{(2)} + C_2 G_D^{(2)}\} \tag{8}$$

where

$$G_Q^{(2)} = G^{(2)}(P,0) \cdot G^{(2)}(0,Q) \tag{9.1}$$

and

$$G_D^{(2)} = C_3 \cdot \mathrm{sign}(\xi) \cdot G_\xi^{(2)}(P,0) + C_4 \cdot G_\eta^{(2)}(P,0) \tag{9.2}$$

It can be shown that the value of the constants C_0 to C_4 can be taken arbitrarily. A comparative study on analytical and numerical methods for treating the irregularities problem has been published in |4|.

Line Integral

The proper evaluation of the free-surface line-integral $I_{F_i}^{(2)}$ (7) is the most critical part in the solution of the second-order diffraction problem. Considering the integrand in (7)

$$i_{F_i}^{(2)} = L_i^{(2)}(\xi,0) \cdot G^{(2)}(x,y,\xi,0) , \tag{10}$$

which has to be evaluated along the free-surface S_F, from its intersection with the body side and up to infinity, the following must be stressed:

- In evaluating $L_i^{(2)}$ (5) the first-order velocity potentials, as well as their first- and second derivatives in x are required along S_F. They can be calculated from $\varphi^{(1)}$, given along S_0 and S_F, through analytical formulas.
- The truncation procedure of $I_F^{(2)}$ at its upper limit, namely at a sufficient large distance away from the body, causes no serious problems, since $i_{F_i}^{(2)}$ is well behaving.
- At the lower limit of $I_F^{(2)}$, namely the intersection of the body side with S_F, the following becomes critical:
  1. The evaluation of $\varphi^{(1)}$ for nonvertical entrances of the body at the waterline, e.g. for "wedge" or "bulb" type sections, causes some trouble. It can be managed sufficiently by the suggested Helmholtz Integral Equation Method.
  2. The evaluation of $\varphi_x^{(1)}$ and $\varphi_{xx}^{(1)}$ requires special care. It can be shown the derivatives of $\varphi^{(1)}$ to possess at the intersection point nonintegrable singularities of the type $O(\xi-b)^{-2}$ resp. $O(\xi-b)^{-3}$ for $\xi = b$. Besides an analytical treatment for it, to be developed in the future, a practical way out of this mess is to assign $\varphi_x^{(1)}$ resp. $\varphi_{xx}^{(1)}$ limiting values according to the horizontal velocity resp. convective acceleration of the body assuming no separation of the fluid at the specific point. The employed formulas for $\varphi_x^{(1)}$ and $\varphi_{xx}^{(1)}$ are :

$$\varphi_x^{(1)} = \sin\alpha\,\varphi_n^{(1)} + \cos\alpha\,\varphi_s^{(1)} \tag{11.1}$$

$$\varphi_{xx}^{(1)} = \cos 2\alpha\,\varphi_{ss}^{(1)} + \sin 2\alpha \cdot (\varphi_{ns}^{(1)} - 0.5\,\rho^{-1}\,\varphi_s^{(1)}) \tag{11.2}$$

where :

$\varphi_n^{(1)} = V_n^{(1)}(s)$: given normal velocity component of the body

$\varphi_s^{(1)} = \frac{\partial}{\partial s}\varphi^{(1)}(s)$: first derivative of $\varphi^{(1)}$ along the arc-length s, evaluated numerically through a cubic spline procedure

$\varphi^{(1)}_{ss} = \frac{\partial^2}{\partial s^2} \varphi^{(1)}(s)$: second derivative of $\varphi^{(1)}$ in s, evaluated numerically from $\varphi^{(1)}_s$

$\varphi^{(1)}_{ns} = \frac{\partial}{\partial s} \varphi^{(1)}_n(s)$: first derivative of $\varphi^{(1)}_n$ in s, evaluated numerically as $\varphi^{(1)}_s$

α : flare angle of the section at the intersection point (vert.:90^0)

$\rho = (x_{ss}y_s - y_{ss}x_s)^{-1}$: sectional curvature at the intersection point

This procedure proved to be satisfactory, as judged from comparisons of calculated wave-diffraction forces with experimental data.

DISCUSSION OF RESULTS

In the present paper special attention is paid to the evaluation of second-order wave forces acting on partially or fully submerged cylinders in waves. Besides some systematic studies for varying cross section shapes, depth of submergence and wave frequency, given partly in |5| and in further reports of the author, herein only comparisons with recent experimental and theoretical data of Kyozuka and Inoue |1| will be commented.

In Figs. 1 to 4, which are reproduced from |1| with the results of the present theory superimposed, the horizontal and vertical steady-state wave drift forces acting on submerging-emerging circular and rectangular cylinders are depicted. For the same conditions the amplitudes of the second-order horizontal and vertical wave forces, namely those oscillating with double the wave frequency, are shown in Figs.5-8. These results are of particular importance, since they involve the proper evaluation of the second-order diffraction problem, outlined before.

The overall agreement of present theoretical-numerical results with the experimental ones of |1| is quite satisfactory, though for depth of submergence to zero the theory seems to fail.

From the physical point of view it is of interest to note the dramatic increase of the nonlinear effects, once a submerged cylinder approaches the free surface, and the change in sign of the vertical drift force, from upwards ("suction effect") to downwards ("squat"), once the cylinder pierces the free-surface. These results are of particular interest when studying the motions of submarines and semi-submersibles in waves.

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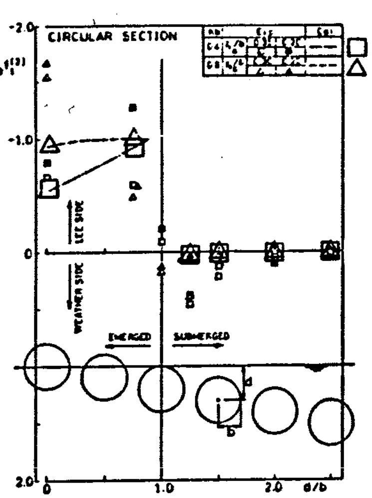

Fig. 1

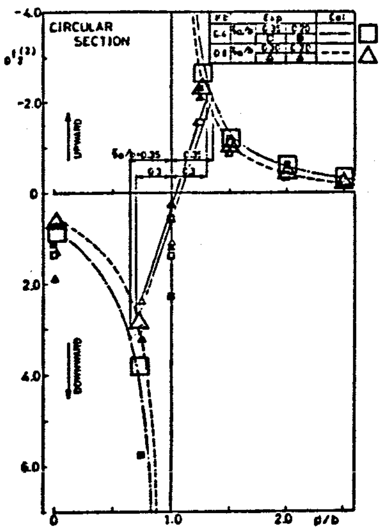

Fig. 2

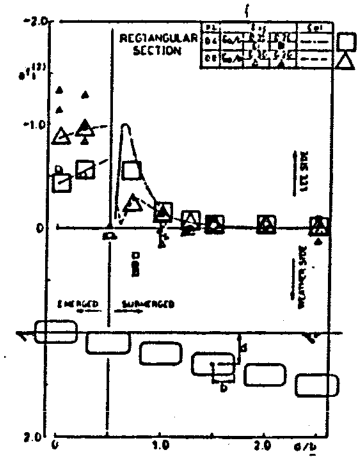

Fig. 3

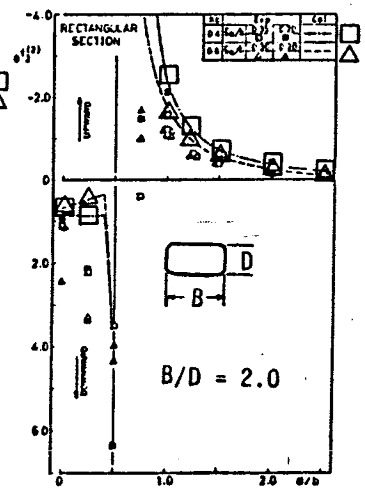

Fig. 4

Figs. 1-4 : Horizontal and Vertical Steady State Wave Forces Acting on Circular and Rectangular Cylinders as a Function of Submergence, Results after |1|, present calculations □ , △ superimposed.

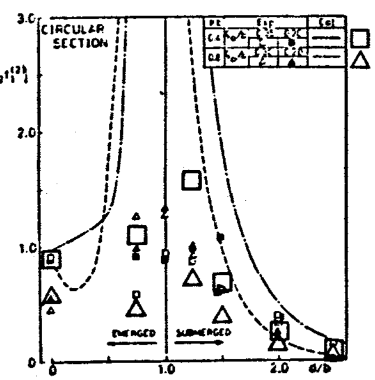

Fig. 5

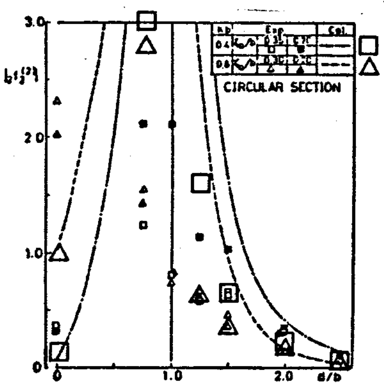

Fig. 6

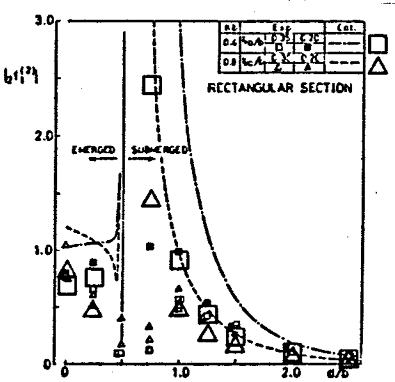

Fig. 7

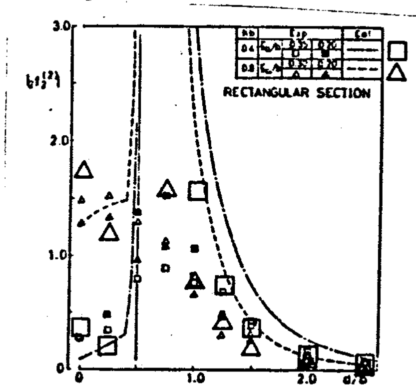

Fig. 8

Figs. 5-8 : Amplitudes of the Second-Order Wave Exciting Forces Acting Horizontally and Vertically on Circular and Rectangular Cylinders as a Function of Submergence, Results after |1|, present calculations □ , △ superimposed.

Discussion

J. Martin: In the diffraction case, how do you deal with the surface line integral whose integrand does not tend to zero as $x \to \infty$? The result should depend on where you truncate the integral and it is not obvious where (within a cycle) the truncation should be made. I question whether it is correct to impose an outgoing radiation condition on the vertical closure in this case, and suggest that the oscillatory surface line integral might be balanced by an oscillatory contribution resulting from the vertical closure.

Papanikolaou: As shown in the oral presentation (and in [5]) the integrand of the line integral oscillates exactly harmonically a few half-beams away from the body so its contribution becomes zero. I have to truncate it but that is not a problem here. Internally, the computer program checks the establishment of an asymptotic behaviour through Haskind's relations.

Yeung: A simple source technique has some advantages over more complex methods. This is because the method generates the solution on the free surface automatically and can be used in second-order problems.

Papanikolaou: I agree. I wish I had known of your method as I started building up my computer program approximately a decade ago. I have to add that the integral-equation method (pulsating sources and dipoles at the body surface, internal source and dipole at the origin and, for the second-order, pulsating sources of known strength at the free surface) works well and is free of irregularities, even for the second-order formulations. I also applied it successfully to extremely flared cross sections. To my knowledge, Kyozuka uses your method at first-order. However, at second-order, he does not solve directly for the second-order velocity potential but employs a line integral to infinity, taken over the free-surface inhomogeneity, to calculate the second-order forces (Soding's 1976 approach).

Evans: A student of mine, J.R. Thomas, in his Ph.D. thesis, looked at mean vertical second order forces on a submerged circular cylinder which was moored by flexible cables. The work was relevant to a wave-energy device being developed at that time. He found that in contrast to the fixed or vertically buoyant free cylinder, the mean vertical force was not always upwards in the constrained case but could at certain frequencies be downwards.

Papanikolaou: In the case of fully submerged circular cylinders in potential flow, Ogilvie (1963) provided analytical expressions for the mean second-order forces in regular waves. In the diffraction case the horizontal drift is trivially zero whereas the vertical drift is non-zero and directed upwards for shallow submergence and goes to zero for deep submergence. In the case of a freely moving cylinder the results are qualitatively the same, but in no case is the mean vertical force directed downwards. My numerical results could have changed the resulting first-order motions so that the resultant mean vertical force becomes downwards. Of course, in the free floating cylinder case the vertical force can be positive or negative depending on the frequency [2].