Evolution of semi-submersible motion in waves

John Martin

Department of Mathematics, University of Edinburgh.

The paper will describe a project, recently commenced, to study the interaction of non-linear waves with twin-cylinder configurations in two dimensions (intended to represent simplified semi-submersible geometry). This is one of a group of projects concerned with stability of semi-submersibles especially in damaged states, aiming ultimately to improve upon the very simple, hydrostatics-based stability regulations which presently apply. Another project in the group will employ linearised regular/random wave and second order calculations. The present work aims to complement this with a numerical time-stepping program to follow the evolution of the motion during the passage of one or two waves of length comparable to pontoon spacing. The formulation will be similar in many respects to existing non-linear evolution programs, but will be optimised for submerged bodies and large but non-breaking waves (i.e. resolution of the advanced stages of jet formation is not an objective).

Non-linear effects are noticeable in model tests of semi-submersibles in quite moderate waves. For instance it has often been observed that, when a twin pontoon model is placed upright in regular beam seas of wavelength approximately twice the pontoon spacing, a steady tilt (almost always to leeward) develops within about two wave periods. The oscillation expected from linear theory then takes place about the tilted mean position. There is some concern that this may be a dangerous contributory factor to listing in damaged states. The related phenomenon of long period (including resonant) second order excitation in random waves is known to contribute significantly to the overall response. Previous work has indicated that steady second order effects are of an appropriate magnitude to influence tilting and that when the first order oscillation and pontoon interaction are properly included, there is a mechanism for a preferred direction of tilt. However, in most cases, appreciable tilting requires high waves and may be limited only by effects outside second order theory. The relevance of the full non-linear (inviscid) theory will be studied using the evolution program.

Only the twin cylindrical pontoons will be considered in the hydrodynamic calculation, the columns and bracing being represented by artificially included hydrostatic restoring terms (there are no problems with free surface intersection points). The following variant of existing methods is being implemented. Spatially periodic conditions are applied in order to avoid difficulty with radiation conditions. Time-stepping is by Hamming's method (applied directly to $\underline{r}$ and ϕ at Lagrangian particles on the free surface). At each time step, the spatial problem is solved by representing the velocity potential ϕ as that of a dipole distribution β on the free surface and a source distribution γ on the pontoons, each with spatially periodic images. The boundary conditions produce coupled Fredholm second kind integral equations for β and γ. Four similar problems for quantities related to $\dot{\phi}$ are also to be solved when body motions are included. In the absence of a body, the equation for β can be solved by iteration, with a simple quadrature rule for the integration. In test cases of initially sinusoidal waves which do not break, convergence is rapid and there is no sign of instability over two (linear) wave periods. In a test case of a wave which breaks, the rate of convergence deteriorates in the later stages of jet formation, but by use of starting approximations obtained by extrapolation from the previous four time steps, accurate results are obtained until well after the time at which the underside of the jet has a horizontal tangent. When a body is present, a convergent iterative scheme is obtained by a modification of the equation on the body boundary. The operator arising from the primary sources (neglecting the periodic images) may be inverted once and for all in body-fixed coordinates. After the inverse operator is applied, it is found in test cases that the resulting system converges at effectively the same rate as the system in the absence of a body. The operator arising from the image-sources is small (of the order of the square of the ratio pontoon dimension to periodic spacing) and is expected to have negligible effect on the local interactions.

Discussion

Kleinman: Even with an eigenvalue of -1, you can use eigenvalue shifting techniques to achieve a convergent iteration. Are all of the eigenvalues the same sign?

Martin: Yes, there are methods to shift eigenvalues and so enable iteration, though to implement them conflicts with the "regularisation" step used in the integration. Eigenvalues of different signs complicate the choice of shift. However, the proposed method is suggested to have other advantages in storage and efficiency.

Yue: According to your extended abstract, how can you invert the body part of the equation system if the body position is changing in time?

Martin: The extended abstract has been revised! The part of the body operator arising from the primary sources (those on the body as opposed to the periodic images) is constant in body-fixed coordinates and can be inverted once and for all. The contribution of the images remains as a "small" operator in the new integral equation.

Papanikolaou: Referring to the "bi-stability" behavior in your presentation, a quasi-linear potential theory (drift forces and moments) will always lead to a tilt moment towards the lee side of the body even for a monohull situation. I would propose to solve the problem in the frequency domain by a perturbation expansion to the second order.

Newman: Have you solved for the second-order moment based on the first-order potential, and what are the results?

Martin: In my earlier work, I solved the first-order problem (with an imposed tilted mean) and computed second-order steady forces and moments in the manner of Ogilvie. Some sample results were shown during the verbal presentation. I found that, for the twin hull configuration, there is a mechanism for a preferred leeward tilt, provided first order oscillation and interaction between the pontoons is properly included. At zero mean tilt, there is a range of wavelengths (round about twice the pontoon spacing) over which the steady moment is to leeward. The actual tilt angles predicted, however, are of a magnitude to put small amplitude theory in doubt.