On the Use of the Rankine Source Potential in the Ship Wave Resistance Problem

by P. S. Jensen 1)

The simple Green function approach in steady-state potential ship wave problems has two serious drawbacks, i.e.:

- The presence of an infinite boundary
- The need for a numerical implementation of the radiation condition.

The present contribution present a solution procedure based on the collocation method using constant source strength and plane elements. By first obtaining the solution analytically it is shown how the radiation condition makes the solution unique.

The radiation condition is in the numerical procedure used in a new way to avoid reflections from the downstream boundary. Physically this corresponds to adjusting an inlet flap at the downstream boundary.

This menas that no damping or artificial viscosity are used in the present procedure.

A more detailed presentation of the present studie can be found in Jensen /1/.

THE BOUNDARY-VALUE PROBLEM

The body is for the present studie assumed thin (3D) or sufficiently submerged (2D).

The boundary-value problem reduces thereby to a Poisson problem where the inhomogeneous part represents the body disturbancy.

THE INTEGRAL FORMULATION

The integral equation for the Poisson problem can be written as:

$$\frac{K}{2}\sigma(\bar{r}) + \iint_S \sigma(\bar{r}_0)\, G\,(|\bar{r}-\bar{r}_0|)ds + f(\bar{r}) = 0 \qquad (1)$$

where S denotes the free surface.

Equation (1) is solved analytically and numerically in both 2D and 3D.

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THE ANALYTICAL SOLUTION

Equation (1) is solved analytically by using Fourier's integral theorem. A singular integrand, corresponding to eigensolutions, forces a transformation of the solution. Complex contour integration is used for this transformation. The fulfilment of the radiation condition determines the integration contour. The transformation transforms the solution into a local part and a part containing waves.

The analytical solution procedure shows that using a simple Green function is not different from using a so-called advanced Green function.

The analytical solution in 2D and in 3D can be found in Jensen /1/.

THE NUMERICAL SOLUTION

The numerical solution of equation (1) is achieved by using the collocation method. The solution domain is divided into two parts:

1) A computational domain, near to the body, where collocation points are distributed, and

2) A far field domain surrounding the computational domain.

The waves in the far field domain are calculated by using the far field approximation of the analytic solution. The discretized version of equation (1) can then be written:

$$\frac{K}{2}\,\sigma_i + \sum_{j=1}^{N} A_{ij}\,\sigma_j = F_i - \sum_{j=1+N}^{N+M} A_{ij}\,\sigma_j \qquad i = 1,\ldots,N \tag{2}$$

where N denotes the number of elements in the computational domain and M is the number of elements in the far field domain.

The coefficients A_{ij} are given in Jensen /1/.

The Radiation Condition

The radiation condition is implemented in two numerically different versions. The first version omits the Poisson condition at the most upstream control point(s). It is instead required that $_k = 0$, $k = 1,..,P$, where k in 2D is the most upstream control point (P = 1) and in 3D the most upstream transverse row (P > 1).

Version No. 2 introduces the radiation condition into all of the N equations. The number of unknowns is then reduced to N-P, where P is defined above. In order to retain a quadratic structure of the system of equations the Poisson boundary condition is omitted at the P most downstream control points. Physically this corresponds to varying a flap angle at the P downstream control points so that waves are not reflected.

It is shown by numerical experiments that the influence of the downstream waves from the far field domain can be neglected when version No. 2 is used. This means that a wall boundary condition can be applied from the flap and to infinity in the downstream direction (c.f. Fig. 1).

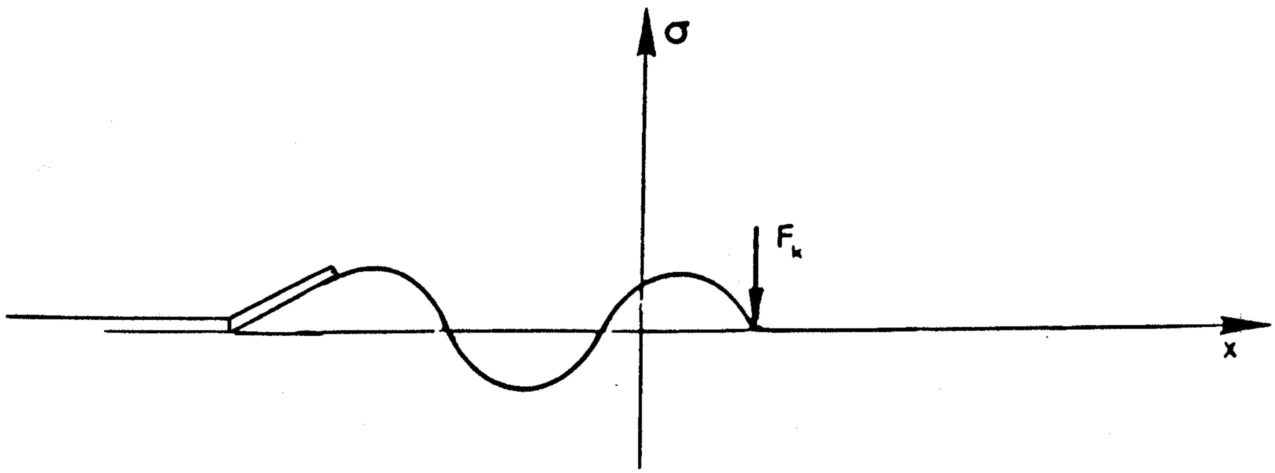

Figure 1.

RESULTS

For the 2D problem results are presented for a fully submerged vortes.

Wave contours and profiles for the Wigley hull will be given and compared with results obtained with the advanced Green function approach.

REFERENCES

/1/ Jensen, P.S., "On the Numerical Radiation Condition in the Steady State Ship Wave Problem", to be published in the Journal of Ship Research.

Discussion

Yeung: I would like to congratulate the speaker for his fine work.

Jensen: Thank you.

Yeung: Are you aware of the work by Dr. Suzuki presented at the 4th International Conference on Numerical Ship Hydrodynamics in Washington D.C. last September? He used Rankine sources on the free surface and body. He also employed a line of submerged Kelvin-like singularities which cancel the waves downstream. The far-field wave amplitude is available immediately after the problem is solved.

Tuck: I would also like to congratulate the speaker and I will be happy to see a written version of the full paper. I think the use of wave absorbers to implement a radiation condition is also relevant if one is using a more complicated Green's function.