Numerical Methods for Nonlinear Two-Dimensional Waves: Regriding Versus Smoothing

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The usefulness of mixed Eulerian-Lagrangian solution schemes as applied to the study of water waves was clearly demonstrated by Longuet-Higgins and Cokelet (1976). The algorithm requires two steps: first, Laplace's equation is solved in a fixed frame of reference, and then Lagrangian points are followed to update the position of the free surface and the potential on the free surface. This method was generalized by Vinje and Brevig (1981) for the case when two-dimensional bodies were present in the fluid. But the intersection of the body with the free surface was not properly treated by Vinje and Brevig. However, Lin (1984) developed a consistent theory for treating the intersection problem.

As an example of the method's application, we consider a piston wavemaker which is forced to surge sinusoidally. In the mixed Eulerian-Lagrangian approach, we first find the solution to Laplace's equation which satisfies Neumann boundary conditions on the wavemaker and Dirichlet boundary conditions on the free surface. Cauchy's integral theorem can be used to solve the field equation. The method of images is used to eliminate the boundaries on the bottom and end of the tank, and Fredholm integral equations of the second kind are obtained for the unknown potential and stream functions on the wavemaker and free surface respectively. Upon solving the equations, the nonlinear boundary conditions on the free surface are used to proceed to the next time step. The solution to Laplace's equation and the time-stepping procedure are by necessity approximations. Nevertheless, the numerical scheme must be accurate and stable.

A point collocation method is used to find the approximate solution to the integral equations, and Adams-Bashforth's predictor-corrector method with a Runge-Kutta starter is used to integrate the boundary conditions on the free surface. In general, forty panels per wavelength and forty time steps per wave period are found to give acceptable accuracy. But in addition to accuracy the stability of the time integrator must also be considered. In fact, sawtooth instabilities have been observed by Longuet-Higgins and Cokelet, and others. Since the instability is not limited to steep waves, it would seem that the linearized equations can give some insight into the nature of the instability.

If a matrix technique is used to find the stability criteria, it can be shown that the time integrator is stable if the transfer function's maximum eigenvalue does not exceed one. The condition for stability prescribes a relationship between the time step and the grid size. For example, for the four-order Runge-Kutta scheme the maximum time step is proportional to the square root of the smallest panel size. In the mixed Eulerian-Lagrangian solution method, the Lagrangian points tend to concentrate in regions of large gradients. Although the increase in accuracy near the cusp of a wave is desirable, this same feature, we believe, also causes the scheme to be inherently instable. Typically, smoothing is used to remove the high frequency noise that is characteristic of the instability. But the tendency to become instable can only be eliminated if the grid size is not allowed to become too small. So every few time steps we find the length of the free surface and divide it into equal segments. Then we interpolate the value of the potential at each of the new grid points and restart the time integration.

Regriding is not new to fluid mechanics. For instance, the regriding scheme of Fink and Soh (1974) extended the range of vortex sheet problems that are possible to do. Yet, despite the successful application of regriding to vortex problems, regriding is not without its disadvantages. For example, grid points will not concentrate near the cusp of a wave where greater accuracy might be required. But the disadvantages are offset by the advantages of regriding:

1. Numerical instabilities arise when regriding is not used because smoothing does not reduce the tendency of mixed Eulerian-Lagrangian solution schemes to become unstable.

2. Regriding can be used to match nonlinear inner and linear outer solutions which are solved using Lagrangian and Eulerian points respectively.

3. Errors caused by regriding decrease as grid size decreases; a smoothing filter always smoothes.

4. Smoothing filters cannot be applied at the intersections of the free surface with the wavemaker.

Furthermore, we believe that regriding would not be required if Eulerian points are followed on the free surface because they do not concentrate like Lagrangian points do. For example, the diffraction of an incident wave about a vertical circular cylinder can be solved using Eulerian points. But Lagrangian points can solve a larger class of problems. In particular, the wavemaker problem is best solved using Lagrangian points. As a result, numerical solutions to the wavemaker problem are used to compare the effects of regriding to smoothing.

The wavemaker is forced to surge sinusoidally for a range of stroke amplitudes. In each case, the effects of regriding are compared to those of smoothing. Specifically, the power expended by the wavemaker should be equal to the rate at which energy is building up in the fluid. For most of the numerical simulations, the power input by the wavemaker relative to that in the fluid is in error by less than four percent in amplitude and five degrees in phase. Moreover, the errors caused by applying regriding and smoothing are comparable for moderately steep waves, but as the wave steepened the errors caused by smoothing became unacceptable. In fact, in one case the smoothing filter was not able to suppress a sawtooth instability and the numerical scheme broke down.

As further evidence of the robustness of regriding, a numerical simulation is compared to the experiments of Chan and Melville (1985). Chan and Melville generated a plunging breaker by varying the frequency content of the wavemaker. Wave probes measured the wave's amplitude, and lasers measured water-particle velocities. The agreement between experiment and theory is satisfactory in the breaking zone.

The numerical experiments substantiate the theory that regriding inhibits the tendency of mixed Eulerian-Lagrangian schemes to become unstable. Whereas smoothing treats the symptoms of instability and not the cause.

References

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Discussion

C. Lee: Did you compare the experiments with a linear or non-linear theory?

D. Yue: A linear solution using a spectral method was used to estimate the effect (reflection) of the far wall of the wavemaker for the nonlinear solutions. All the comparisons with experiments we show are for the nonlinear theory.

Papanikolaou: (1) Did you compare your theory with the second-order forces on a heaving cylinder in the frequency domain?

(2) When imposing boundary conditions on the body and free surface, you said the body boundary conditions are more important. I did calculations in the frequency domain where I omitted one or both, depending on the mode and shape. I found that neither one can be omitted; one is as important as the other.

Dommermuth: We compared the circular cylinder results with a linear second-order spectral solution and the agreement was fair. Molin would agree that this is due to the difficulty in integrating the Bernoulli term in the pressure around the sharp corner at the cylinder's base. However, we found excellent agreement for the cone with frequency-domain results.

Papanikolaou: I was asking about second-order forces on a heaving cylinder after it reaches steady state.

Yue: We computed higher-order forces by taking a period of the nonlinear time-domain solution after steady state is reached and then applying a Fourier analysis. The agreement was fair for the cylinder, but within a few percent accuracy for the cone.

Newman: Could you describe how the numerical simulation of the wave-tank experiments was accomplished?

Dommermuth: We were given the time history of the displacement of the wavemaker from the experiment and simply used the same time history to do the numerical calculation.