Nonlinear Resonance of Long Shelf Waves by Incident Short Waves

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Summary

It is well known that shallow water waves can be trapped on a shelf or on a ridge (or outside a canyon). These waves must be propagating on the ridge in a direction oblique to its axis. It is impossible by any linear mechanism for these waves to have entered the ridge from the deeper sea; they must be generated by local forcing on the ridge. The wave length and period of these trapped modes are much longer than those of swell with typical periods of O(10 sec), but can be close to the modulational frequency of swell groups,typically a few minutes.

We shall show that swell groups incident from the sea can excite trapped long waves on a shelf. If the excitation is resonant, these waves can be greater than second order, and cause large slow oscillations. Hence these long waves may present a hazard to off-shore platforms.

The subject of interaction of slowly varying waves with topography, and trapping on a shelf was studied by Foda and Mei (1982) and by Mei and Benmoussa (1984) who give references to other work. The latter authors studied a slowly varying topography in which reflection by the sloping bottom is negligible. In the present work we study the case when reflection of the short waves is significant.

For demonstration of the physics, we choose a relatively simple bathymetry in the form of a rectangular step. The width of the shelf is comparable to the wave group length, which is much greater than the short wave wavelength. The water depths are comparable to the short wave length. To solve for the flow potential, we perform a perturbation expansion in terms of the short wave steepness, and utilize multiple time and (horizontal) space scales.

At the leading order, the short waves are found by the solutions of a linear diffraction problem studied by Mei and Black (1969) using a variational method - or, equivalently, by Galerkin method. For convenience, the problem is solved as a superposition of a symmetric problem and an antisymmetric one. In contrast to the procedure employed for a floating body (Agnon and Mei (1985)), the reflection of the shelf is very sensitive to the short wave wavelength. This is because the shelf is long; even small variations within a narrow banded spectrum give rise to large phase differences that affect reflection. An alternative and equivalant approach is to break down the shelf into two steps, matching their far fields (cf. Newman (1965)) while retaining the dependence of the wave amplitude on the slow (stretched) time.

After finding the short wave field, we examine separately the near field within a few short waves of the depth steps, and the far field, a few wave groups away. We find the forms of the potentials for the slow waves in each field, and match their asymptotic values to get the solution. It is found that two types of long waves exist. The first one is locked to the short-wave groups and propagates at the group velocity. The other is a shallow water wave generated at the shelf edges and radiated away from them in directions different from those of the short waves and the locked long waves.

When the depths and angle of incidence are such that the second long wave is trapped on the shelf, resonance can occur. The resonance obtained from the present approximation is unbounded, since there is no radiation damping due to complete trapping of the long waves. The onset of resonance, however, takes a very long time.

We present results for the amplitude of the free long waves due to periodic and transient inputs short waves incident normally or obliquely.